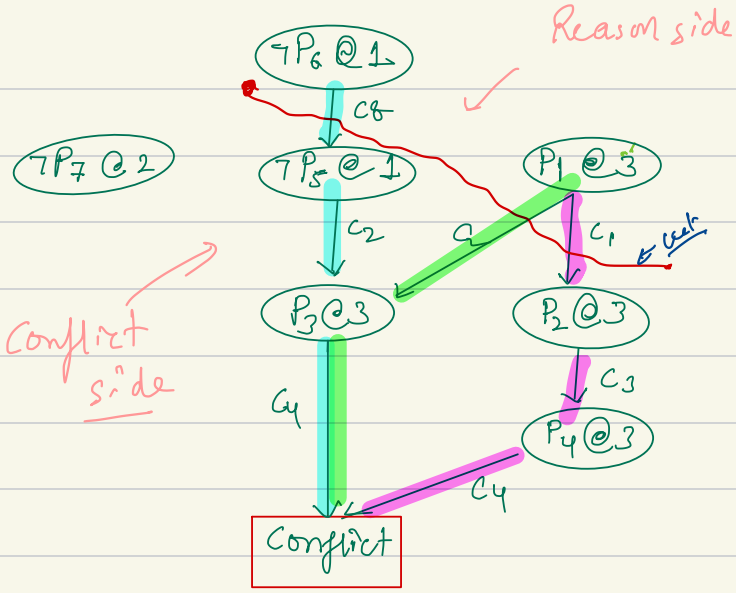
CDCL

1. Select a variable and assign randomly True or False
2. Do unit propagation
3. Build the implication graph
4. If there is a conflict
   - → Find the **cut** in the implication graph that led to the conflict.
   - → Drive a **new clause** (conflict clause) : negation of assignments that led to conflict.
   - → **Non chronologically Backtrack** to appropriate level
5. Otherwise continue with step 1, until all variables are assigned.



conflict clause = $\neg(\neg P_6 \wedge P_1)$

$P_6 \vee \neg P_1$

(there are other clever choices for conflict clause)

Backtrack based on conflict clause: Backtrack to decision level m, where m be the second largest decision level of the literals in conflict clause "C" (or '0' if C contains only one literals).

Conflict Clause

In case of conflict, we traverse the implications graph backwards to find the set of decision that caused conflict.

Adding conflict clause

1. doesn't change the set of satisfying assignments.
2. implies that conflicting partial assignment will never be tried again.

Prunes search space

Records past work of solver

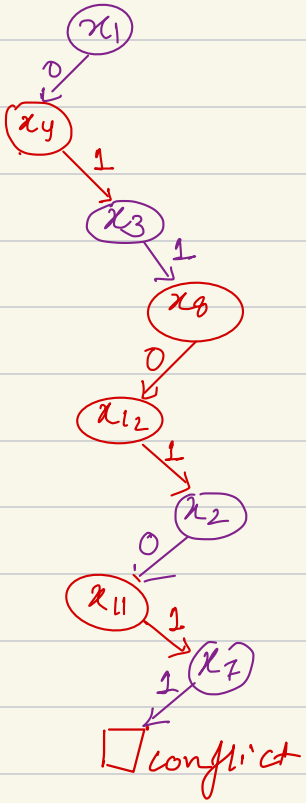
Multiple clauses can satisfy the above two conditions.

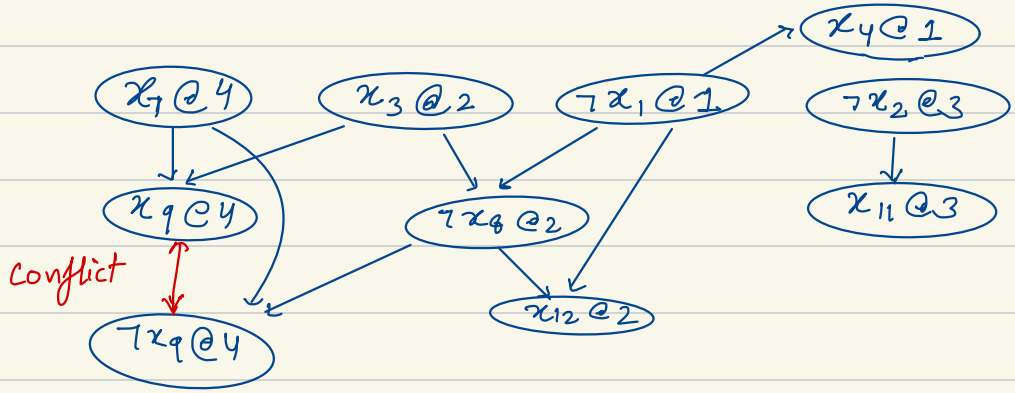
$F: (x_1 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_8) \wedge (x_1 \vee x_8 \vee x_{12}) \wedge (x_2 \vee x_{11}) \wedge$
$(\neg x_7 \vee \neg x_3 \vee x_9) \wedge (\neg x_7 \vee x_8 \vee \neg x_9) \wedge (x_7 \vee x_8 \vee \neg x_{10}) \wedge (x_7 \vee x_{10} \vee \neg x_{12})$

order $x_1, x_3, x_2, x_7, x_5, x_8, x_9, x_{12}, x_{11}$

0 1 0 1 0 1 0 1 0

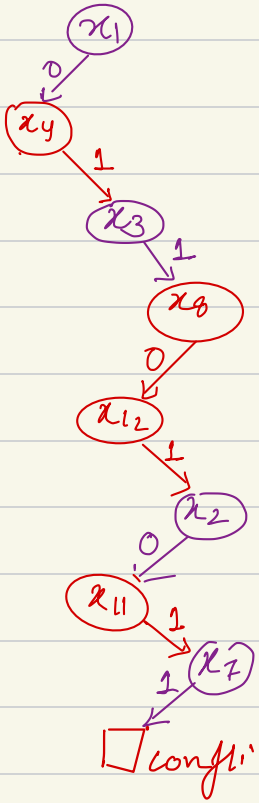
$F: (x_1 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_8) \wedge (x_1 \vee x_8 \vee x_{12}) \wedge (x_2 \vee x_{11}) \wedge$
$(\neg x_7 \vee \neg x_3 \vee x_9) \wedge (\neg x_7 \vee x_8 \vee \neg x_9) \vee (x_7 \vee x_8 \vee \neg x_{10}) \wedge (x_7 \vee x_{10} \vee \neg x_{12})$

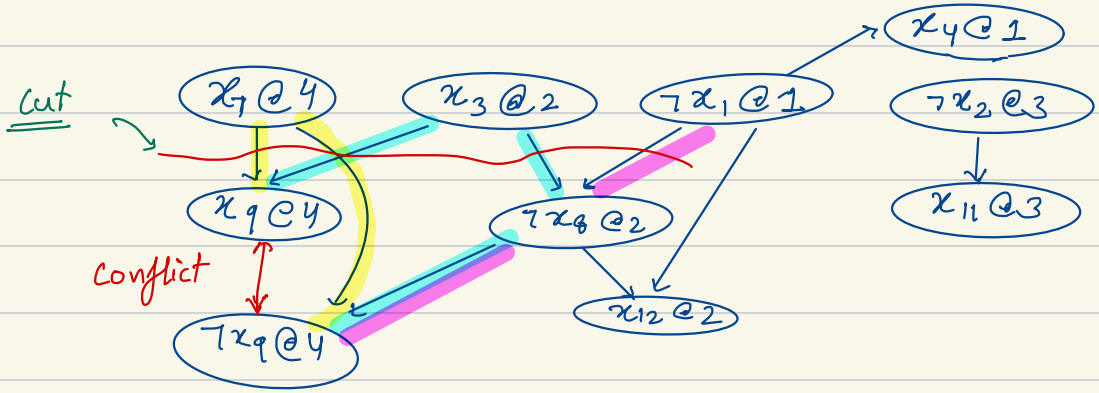




Implication graph.

$F: (x_1 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_8) \wedge (x_1 \vee x_8 \vee x_{12}) \wedge (x_2 \vee x_{11}) \wedge$
$(\neg x_7 \vee \neg x_3 \vee x_9) \wedge (\neg x_7 \vee x_8 \vee \neg x_9) \vee (x_7 \vee x_8 \vee \neg x_{10}) \wedge (x_7 \vee x_{10} \vee \neg x_{12})$



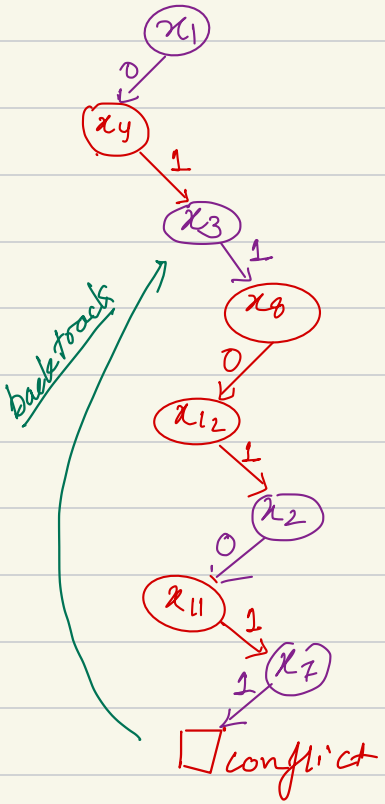


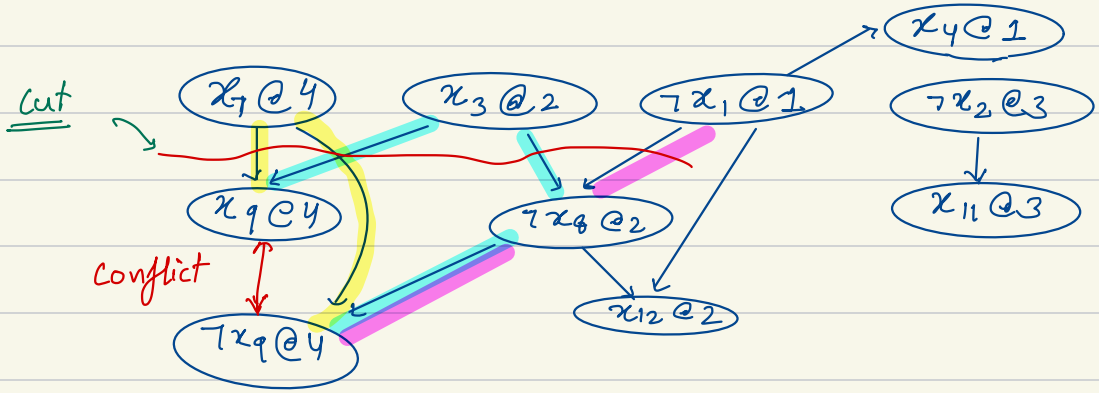
one of the possible cut

learned clause = $\neg x_7 \vee \neg x_3 \vee x_1$

Backtrack to level 2

$F: (x_1 \vee x_4) \wedge (x_1 \vee \neg x_3 \vee \neg x_8) \wedge (x_1 \vee x_8 \vee x_{12}) \wedge (x_2 \vee x_{11}) \wedge$
$(\neg x_7 \vee \neg x_3 \vee x_9) \wedge (\neg x_7 \vee x_8 \vee \neg x_9) \vee (x_7 \vee x_8 \vee \neg x_{10}) \wedge (x_7 \vee x_{10} \vee \neg x_{12})$





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Backtrack to level 2

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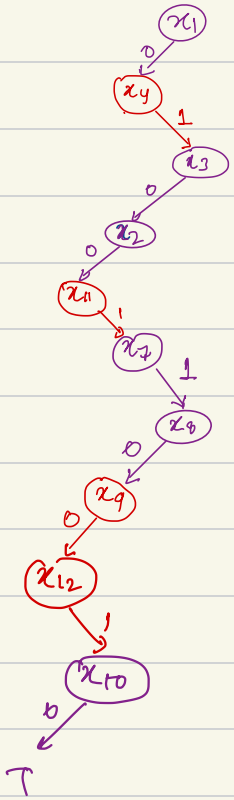
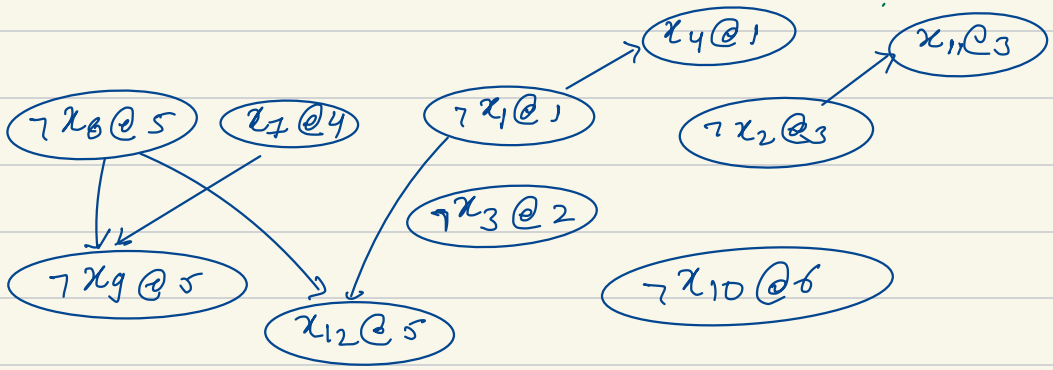




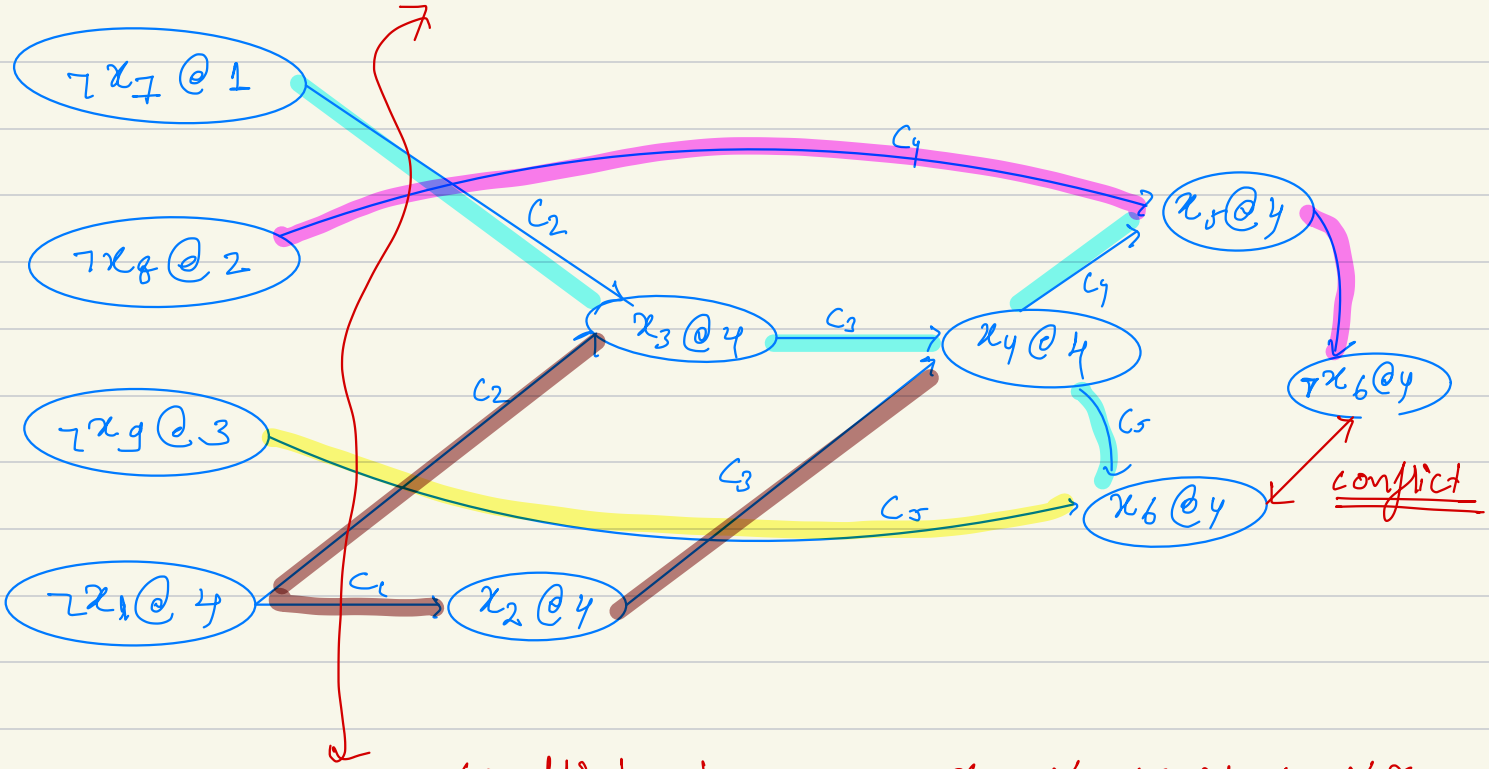
$$F = (x_1 \vee x_2) \wedge (x_1 \vee x_9 \vee x_7) \vee (\neg x_2 \vee \neg x_3 \vee x_4) \vee$$
$$(\neg x_4 \vee x_5 \vee x_8) \vee (\neg x_4 \vee x_6 \vee x_9) \vee (\neg x_5 \vee \neg x_6)$$

Order

$\langle x_7, x_8, x_9, x_1, x_2, x_4, x_5, x_3, x_6 \rangle$

polarity always 0.

$x_7 = 0$, $x_8 = 0$, $x_9 = 1$, $x_1 = 0$, $x_2 = 1$, $x_3 = 1$,
$x_4 = 1$, $x_5 = 1$, $x_6 = 0$



conflict clause = $x_7 \vee x_8 \vee x_9 \vee x_1$

Backtracking to level 3, change polarity of x_9

choices for conflict clauses

- Smaller conflict clauses prune more search space -

1. Cuts in the implication graphs
2. 1-UIP
   - Unique implication point