COL:750

Foundations of Automatic Verification

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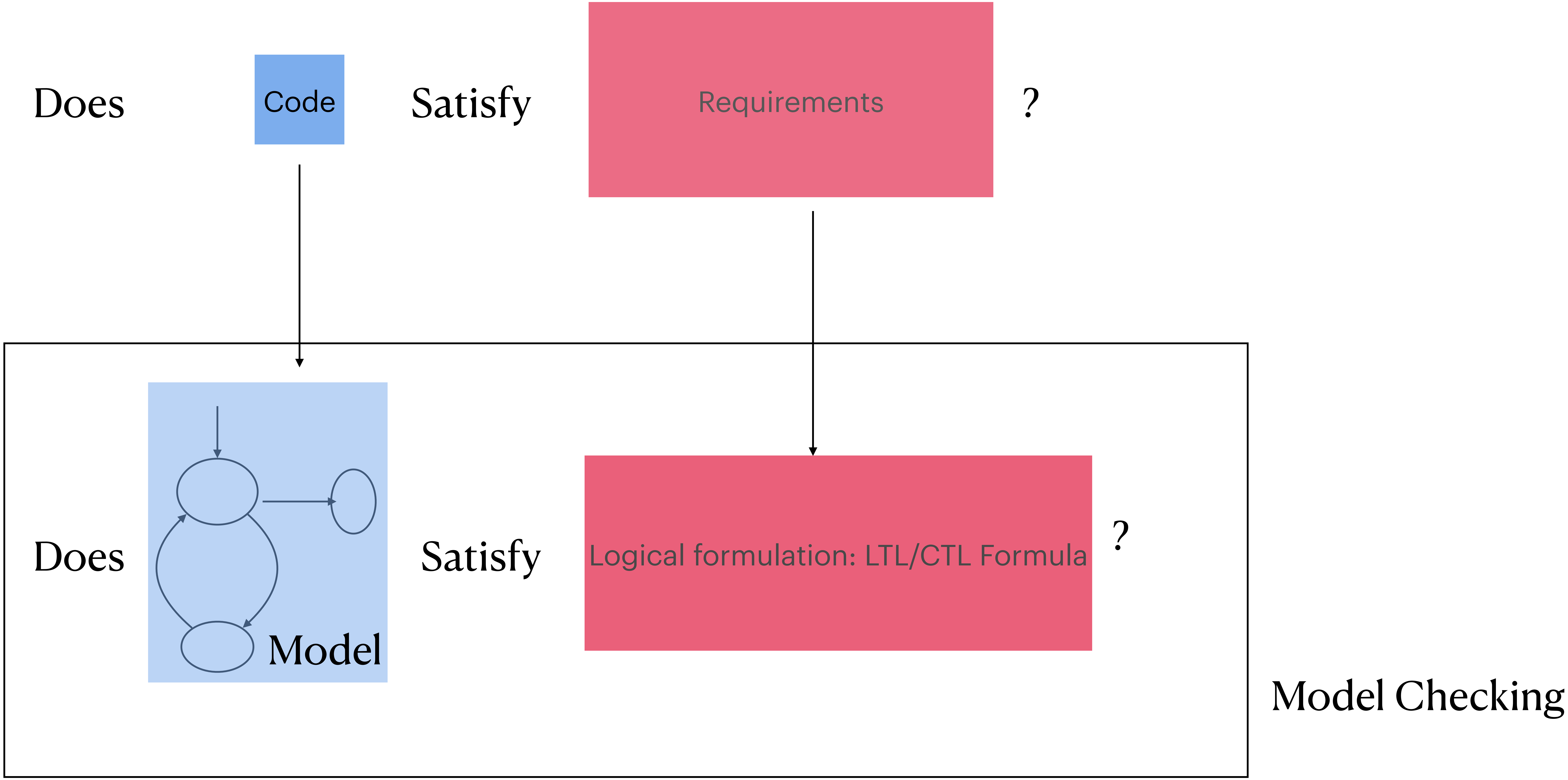
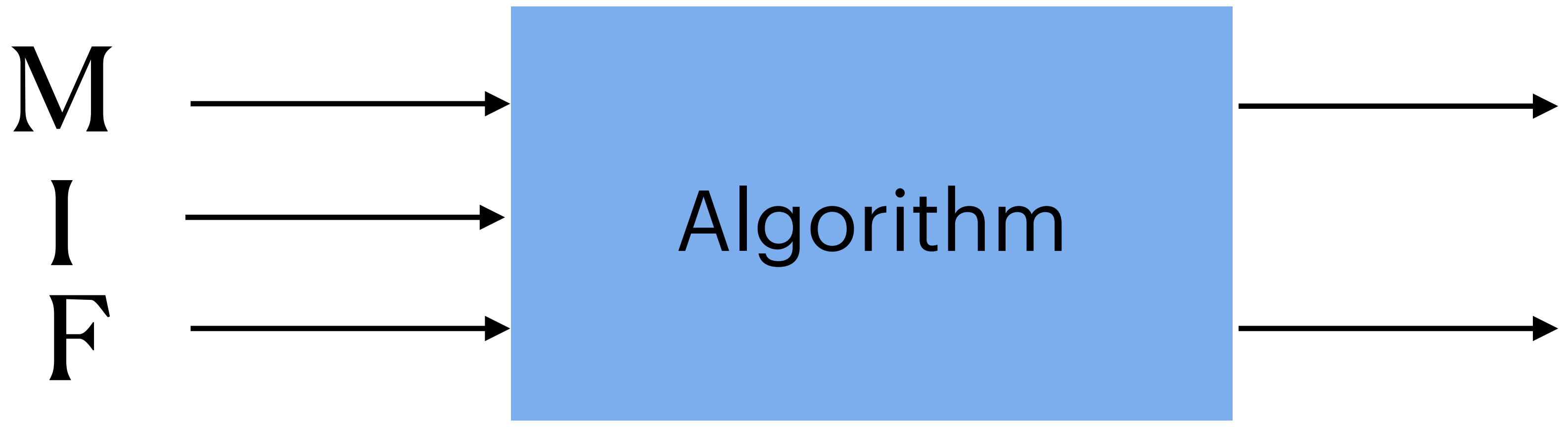
Course Webpage

https://priyanka-golia.github.io/teaching/COL-750/index.html


Does
Code
Satisfy
Requirements
?
Does
Model
Satisfy
Logical formulation: LTL/CTL Formula
?
Model Checking

Model Checking Algorithm

$$M, s \models F?$$



Yes, if $M, s \models F, \forall s \in I$

No, if $\exists s \in I \ s.t.\ M, s \not\models F$,

A path of the system M demonstrating that M can't satisfy F

Model Checking Algorithm

$$M, s \vDash F?$$

$S' \quad s.t.\ S' \subseteq S$, and $M, s \vDash F\ \forall s \in S'$

All states s of the model M that satisfy F

Note that not necessarily $I \subseteq S'$

CTL Model Checking Algorithm — Labelling Algorithm

1. Input — a Model M, and a CTL formula F.
2. Output — S' (the set of states of M that satisfy the formula F.)

Key idea — build from sub-formulas.

1. Label the states of M with the sub-formulas of F that are satisfied at the state.
2. Starting with smallest sub formula and working towards F.

$$F = (\forall \Box (p \rightarrow \forall \Diamond q))$$

$q \qquad \forall \Diamond q \qquad p \qquad p \rightarrow \forall \Diamond q \qquad \forall \Box (p \rightarrow \forall \Diamond q)$

CTL Model Checking Algorithm — Labelling Algorithm

1. Input — a Model M, and a CTL formula F.
2. Output — S' (the set of states of M that satisfy the formula F.)

Key idea — build from sub-formulas.

1. Constructs the set of states where the formula holds:
$[F] := s \in S$ **s.t.** $M, s \vDash F$
2. Then, compare the $[F]$ with the set of initial states : $I \subseteq [F]$?

Compute $[F]$ in "bottom-up" on the structure of formula — computing $[F_i]$ for each sub-formula F_i of F.

$F = (\forall \Box (p \rightarrow \forall \Diamond q))$ $\quad q \quad \forall \Diamond q \quad p \quad p \rightarrow \forall \Diamond q \quad \forall \Box (p \rightarrow \forall \Diamond q)$

CTL Model Checking Algorithm — Labelling Algorithm

Compute $[F]$ in "bottom-up" on the structure of formula — computing $[F_i]$ for each sub-formula F_i of F.

How to compute $[F_i]$?

Case Analysis

Recall given $M := < S, I, R, L >$

$\bot$ — no states are labelled with $\bot$ $[\bot] = \{\}$

P — label s with p if $p \in L(s)$ $[p] = \{s \mid p \in L(s)\}$

$\neg F_1$ — label s with $\neg F_1$ if s is NOT already labelled with F_1 $[\neg F_1] = S \backslash [F_1]$

$F_1 \wedge F_2$ — label s with $F_1 \wedge F_2$ if s is already labelled both F_1 and F_2

$[F_1 \wedge F_2] = [F_1] \cap [F_2]$

CTL Model Checking Algorithm — Labelling Algorithm

Case Analysis

Recall given $M := < S, I, R, L >$

$\exists \mathbf{N} F_1$

If any state s is labelled with F_1 if one of its successor is labelled with F_1

$$[\exists \mathbf{N} F_1] = \{s \in S \mid \exists s' < s, s' > \in R \wedge s' \in [F_1]\}$$

$[\exists \mathbf{N} F_1]$ is called pre-image of $[F_1]$ (pre($[F_1]$))

CTL Model Checking Algorithm — Labelling Algorithm

Case Analysis

Recall given $M := < S, I, R, L >$

$\exists \Box F$

$\Box F \equiv F \wedge \mathbf{N}(\Box F)$

$\exists \Box F \equiv F \wedge \exists \mathbf{N}(\exists \Box F)$

Label any state with $\exists \Box F_1$ if

1. it is labelled with F_1 and one of its successor is labelled with $\exists \Box F_1$ until there is no change.

$[\exists \Box F] = [F] \cap pre([\exists \Box F])$

CTL Model Checking Algorithm — Labelling Algorithm

Case Analysis

Recall given $M := < S, I, R, L >$

$\exists \Box F$

$\Box F \equiv F \wedge \mathbf{N}(\Box F)$

$\exists \Box F \equiv F \wedge \exists \mathbf{N}(\exists \Box F)$

$[\exists \Box F] = [F] \cap pre([\exists \Box F])$

We can compute this inductively.

$X_1 = [F_1]$

$X_2 = X_1 \cap pre(X_1)$

..

$X_{j+1} = X_j \cap pre(X_j)$

Since $X_{j+1} \subseteq X_j$ for every $j \geq 0$, thus a fix point always exists.

CTL Model Checking Algorithm — Labelling Algorithm

Case Analysis

Recall given $M := < S, I, R, L >$

$\exists F_1 \mathbf{U} F_2$

$F_1 \mathbf{U} F_2 \equiv F_2 \vee (F_1 \wedge \mathbf{N}(F_1 \mathbf{U} F_2))$

$\exists F_1 \mathbf{U} F_2 \equiv F_2 \vee (F_1 \wedge \exists \mathbf{N} \exists (F_1 \mathbf{U} F_2))$

Label any state with $\exists F_1 \mathbf{U} F_2$ if

1. it is labelled with F_2, or
2. it is labelled with F_1 and one of its successor is labelled with $\exists F_1 \mathbf{U} F_2$

until there is no change.

$$[\exists F_1 \mathbf{U} F_2] = [F_2] \cup ([F_1] \cap pre([\exists (F_1 \mathbf{U} F_2)]))$$

CTL Model Checking Algorithm — Labelling Algorithm

Case Analysis

Recall given $M := < S, I, R, L >$

$\exists F_1 \mathbf{U} F_2$

$F_1 \mathbf{U} F_2 \equiv F_2 \vee (F_1 \wedge \mathbf{N}(F_1 \mathbf{U} F_2))$

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We can compute this inductively.

$X_1 = [F_2]$

$X_2 = X_1 \cup ([F_1] \cap pre(X_1))$

..

$X_{j+1} = X_j \cup ([F_1] \cap pre(X_j))$

Since $X_{j+1} \supseteq X_j$ for every $j \geq 0$, thus a fix point always exists.

CTL Model Checking Algorithm — Labelling Algorithm

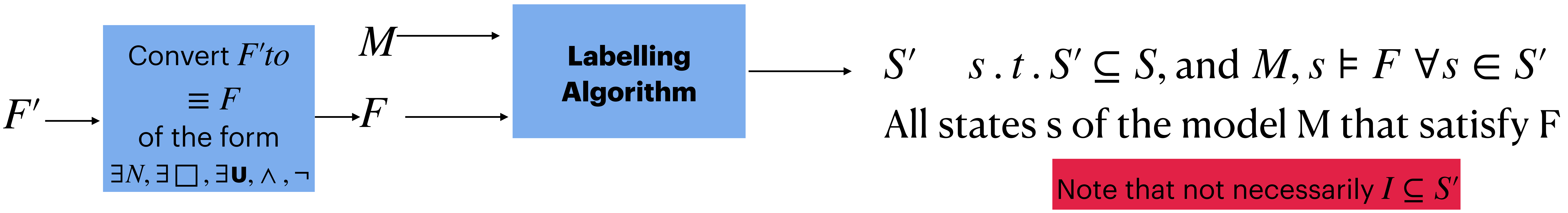
Binary operator $\bot, \neg, \wedge$

Temporal operator $\exists \mathbf{N}, \exists \Box, \exists \mathbf{U}$

Form an adequate set of CTL formulas.

Any given CTL formula can be converted to equivalent CTL formula using only these operator.

$$M, s \vDash F?$$



CTL Model Checking Algorithm — Labelling Algorithm

Function Label$(F, M)\{$

Case F *of* :

True	return S
False	return {}
p	return $\{s \in S \mid p \in L(s)\}$
$\neg F_1$	return $S \backslash Label(F_1)$
$F_1 \wedge F_2$	return $Label(F_1) \cap Label(F_2)$
$\exists \mathbf{N} F_1$	return pre$(Label(F_1))$
$\exists \square F_1$	return $Label\_EG(Label(F_1))$
$\exists F_1 \mathbf{U} F_2$	return $Label\_EU(Label(F_1), Label(F_2))$

End Case

CTL Model Checking Algorithm — Labelling Algorithm

$[\exists \mathbf{N} F_1] = pre([F_1])$

$[\exists \mathbf{N} F_1] = \{s \in S \mid \exists s' < s, s' > \in R \wedge s' \in [F_1]\}$

pre($[F_1]$) {

X = {}

For each s' in $[F_1]$ do:

For each s in S do:

If $< s, s' > \in R$:

$X = X \cup s$

Return X

}

CTL Model Checking Algorithm — Labelling Algorithm

$[\exists \Box F_1]$ $[\exists \Box F] = [F] \cap pre([\exists \Box F])$

$Label\_EG([F_1])$ {

X = $[F_1]$
Y = {}

While $X \neq Y$ do:

Y = X

$X = X \cap pre(X)$

Return X

}

CTL Model Checking Algorithm — Labelling Algorithm

$[\exists F_1 \mathrm{U} F_2]$

$[\exists F_1 \mathbf{U} F_2] = [F_2] \cup ([F_1] \cap pre([\exists(F_1 \mathbf{U} F_2)]))$

$Label\_EU([F_1], [F_2])$ {

X = $[F_2]$
Y = S

While $X \neq Y$ do:

Y = X

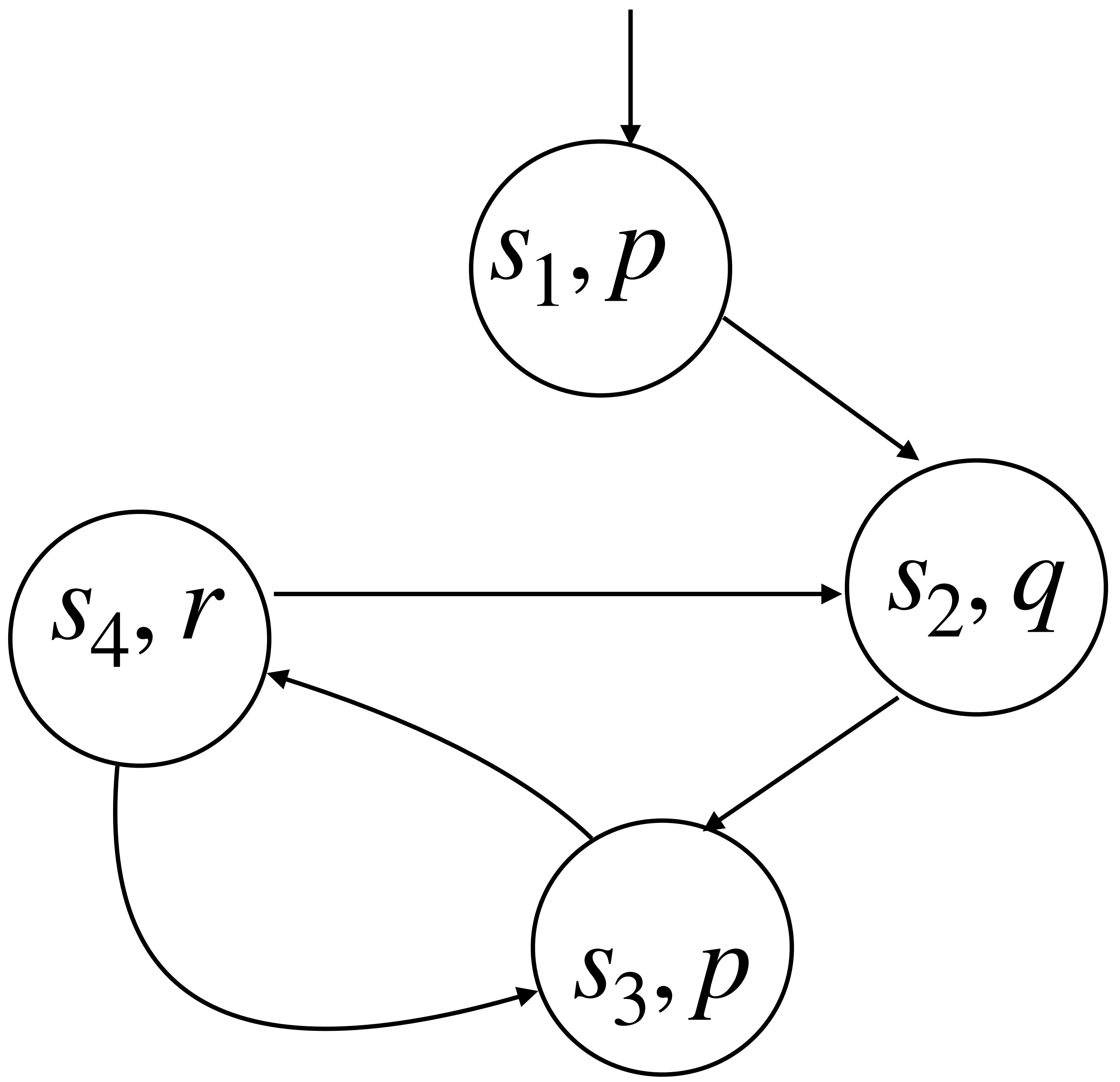
$X = X \cup ([F_1] \cap pre(X))$

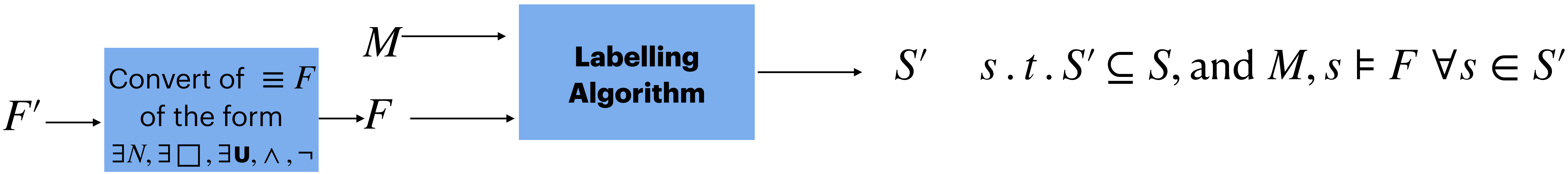
Return X

}

Labelling Algorithm — Example

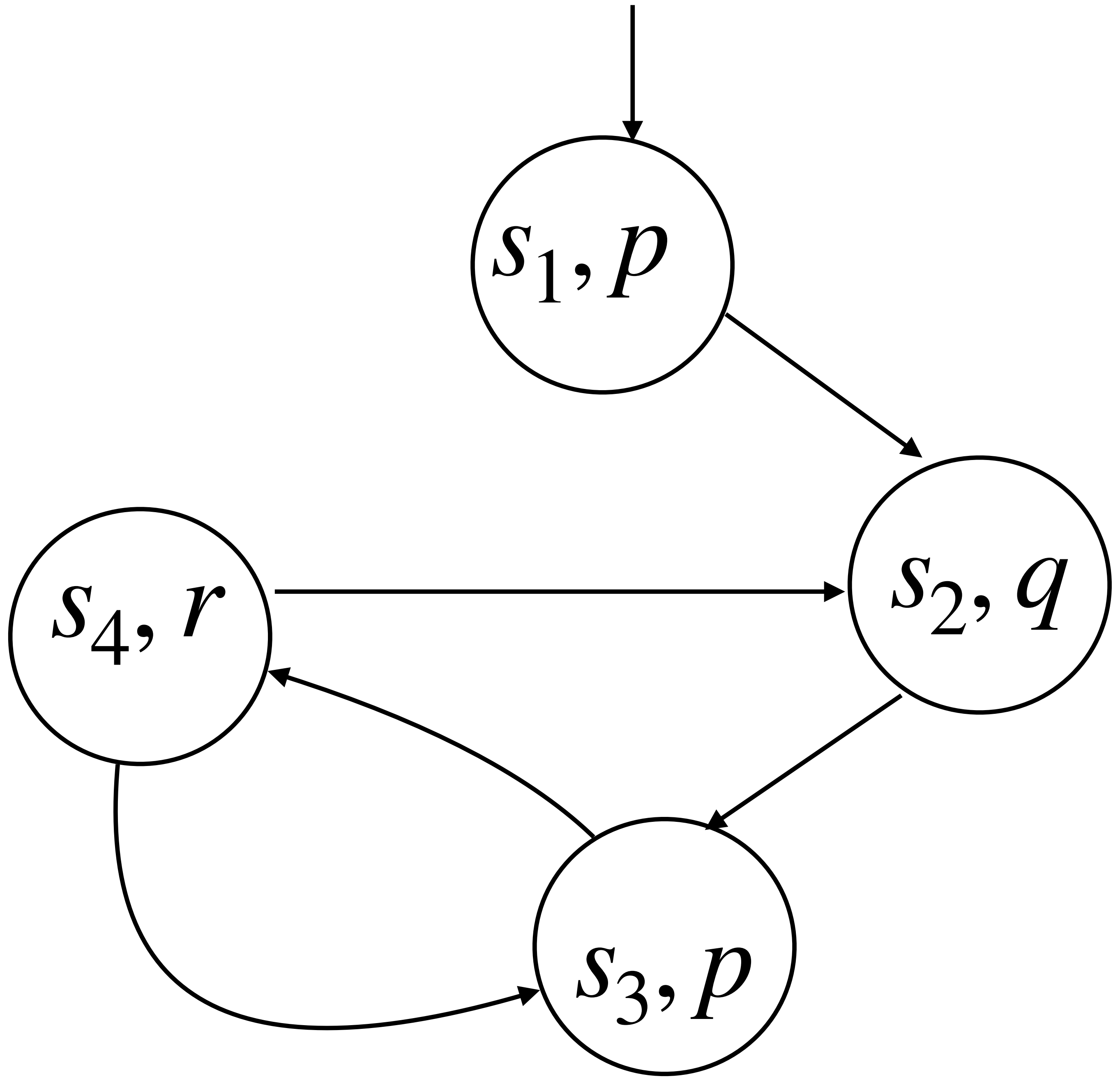
$$F' = (\forall \Box (p \rightarrow \forall \Diamond q))$$





Labelling Algorithm — Example

$$F' = (\forall \Box (p \rightarrow \forall \Diamond q))$$



$$\neg F' = \exists \Diamond \neg (p \rightarrow \forall \Diamond q)$$

$$\neg F' = \exists \Diamond (p \land \neg (\forall \Diamond q))$$

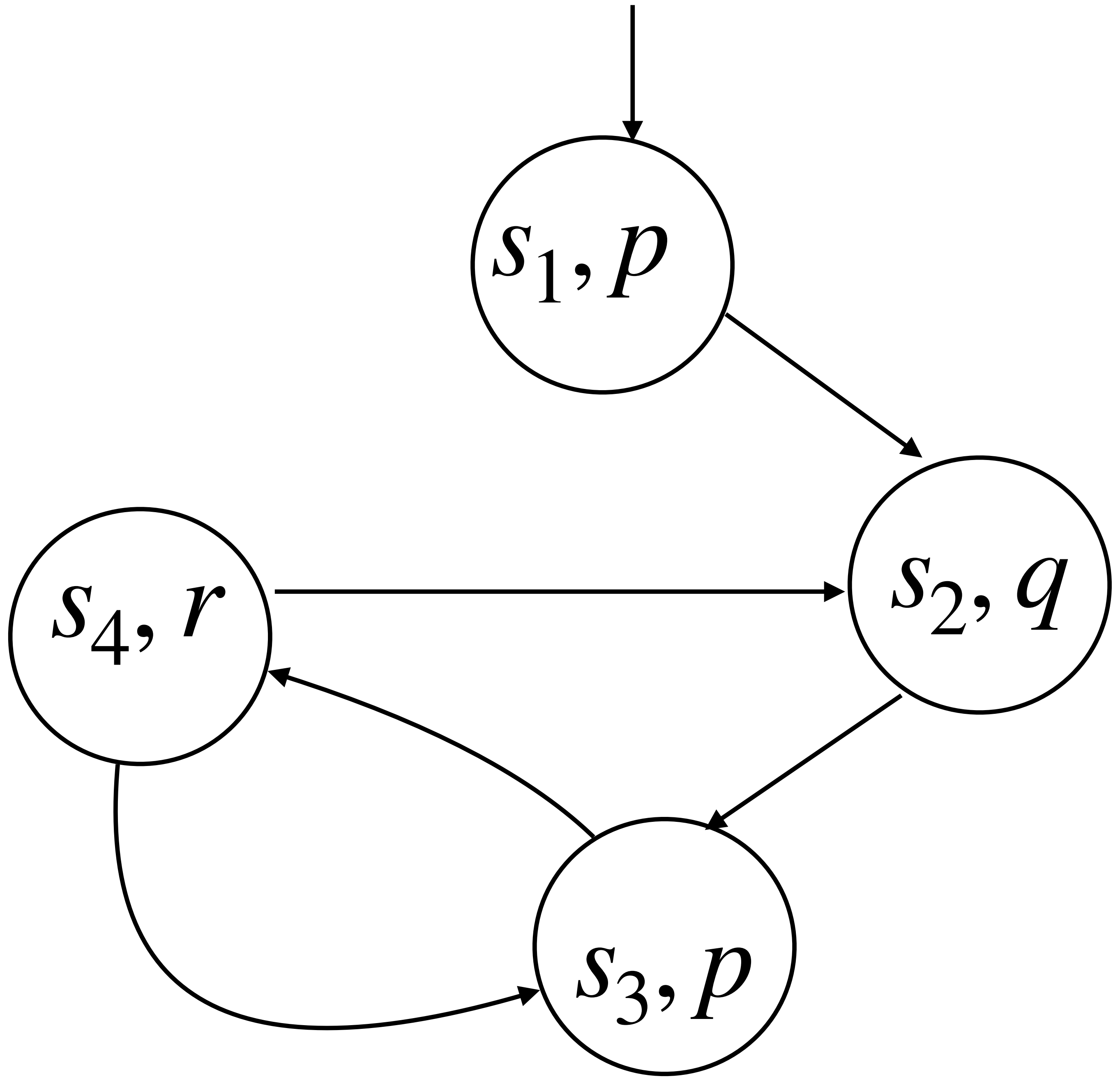
$$\neg F' = \exists \Diamond (p \land \exists \Box \neg q))$$

$$\neg F' = \exists (True\ \mathbf{U}\ (p \land \exists \Box \neg q)))$$

Labelling Algorithm — Example

$\neg F' = True\ \mathbf{U}\ (p \land \exists \Box \neg q))$

$F = \neg(\neg F')$



[True] = $\{1,2,3,4\}$

$[\neg q] = \{1,3,4\}$

$[\exists \Box \neg q] = \{3,4\}$

$[p] = \{1,3\}$

$[p \land \exists \Box \neg q] = \{3\}$

$[True\mathbf{U}(p \land \exists \Box \neg q)] = \{1,2,3,4\}$

$[\neg F'] = \{1,2,3,4\}$

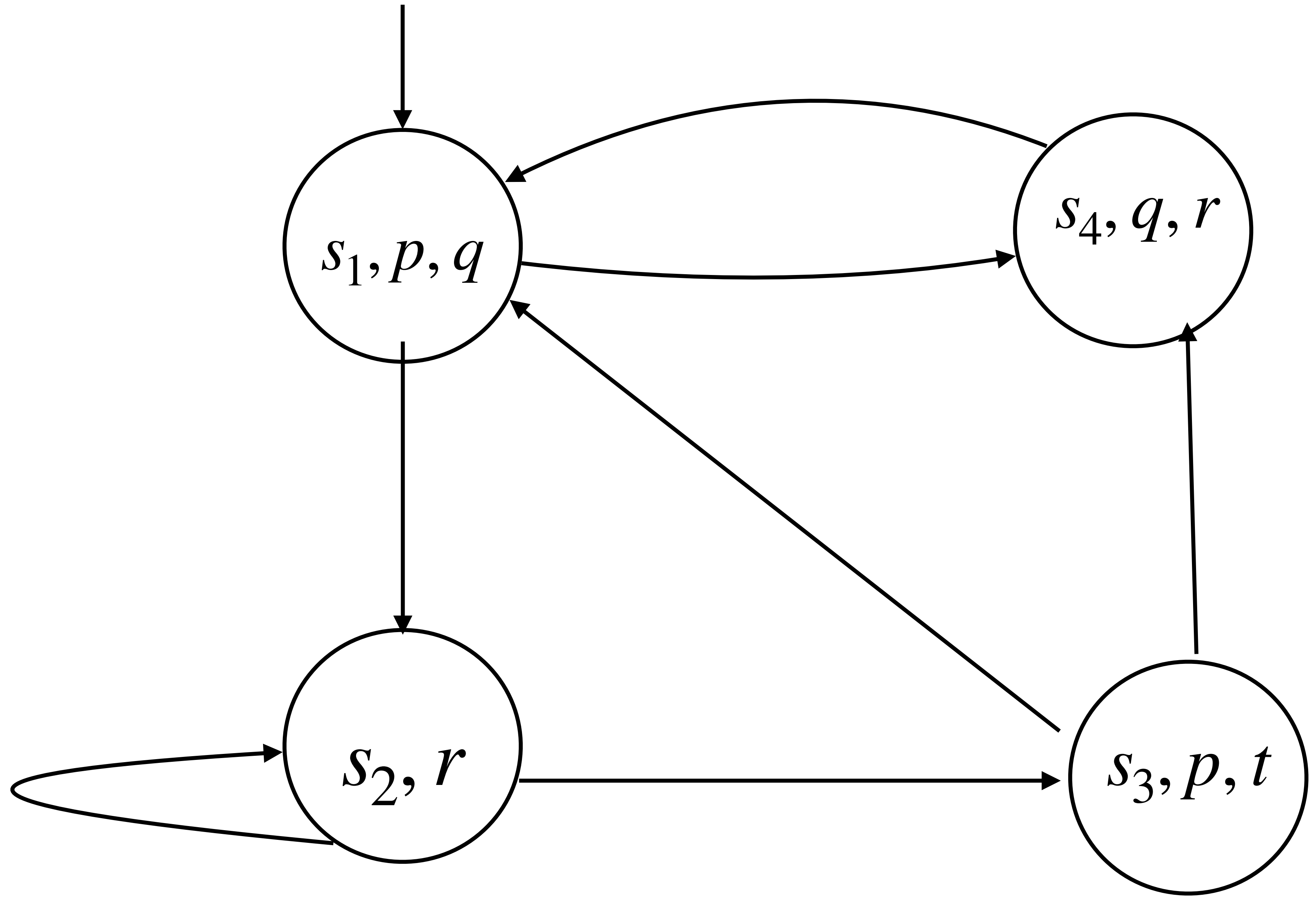
[F] = $\{\}$

Labelling Algorithm — Example

$$F = \forall \square (\forall \lozenge q)$$

$$R = \mathbf{NN}(r)$$

$$S = \forall \square (\exists \lozenge (p \lor r))$$



Exercise: find out the complexity of the labelling algorithm in terms of
number of connectives of the formula (say f).
number of states of the model (say |S|)
number of transitions in the model (say |R|).

It will be linear in f, and quadratic in M

How to improve *Label_EG* algorithm?

Recall strongly connected components.

1. Restrict the graph to states satisfying F_1.
2. Find the maximal strongly connected components (SCC)
3. Use backward breadth first search on the restricted graph to find any state that can reach SCC.