COL:750

Foundations of Automatic Verification

Instructor: Priyanka Golia

Course Webpage

https://priyanka-golia.github.io/teaching/COL-750/index.html

CTL :Examples

Safety: "something bad will never happen"

$$\neg(\exists \Diamond p) \equiv \forall \Box \neg p$$

Reactor_temp is never going to be above 1000.

$$\forall \Box \neg(ReactorTemp > 1000)$$

If car takes left, then immediately car should not take right.

$$\forall \Box \neg(left \ \wedge \ \forall \ \mathbf{N} \ right)$$

CTL :Examples

Liveness: “something good will happen”

$$\forall \Diamond p$$

All students will get their degree

$$\forall \Diamond (Student \wedge degree)$$

If you start something you will eventually finish it.

$$\forall \Box (start \rightarrow \forall \Diamond Finish)$$

CTL : Formula Equivalence

The formulae F_1, F_2 are said to be semantically equivalent if any state in any model that satisfies one also satisfies the other.

$$F_1 \equiv F_2$$

$$\exists \Diamond (a \wedge b) \equiv \exists \Diamond a \wedge \exists \Diamond b$$

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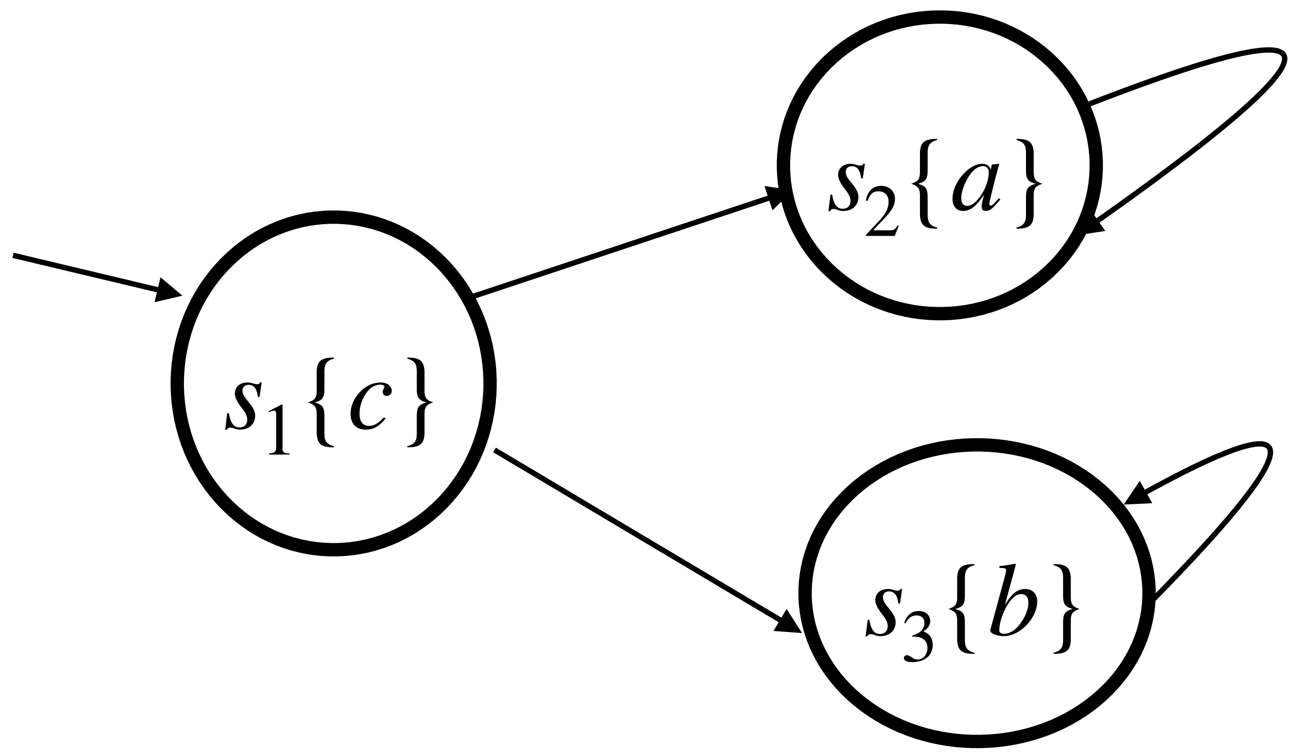

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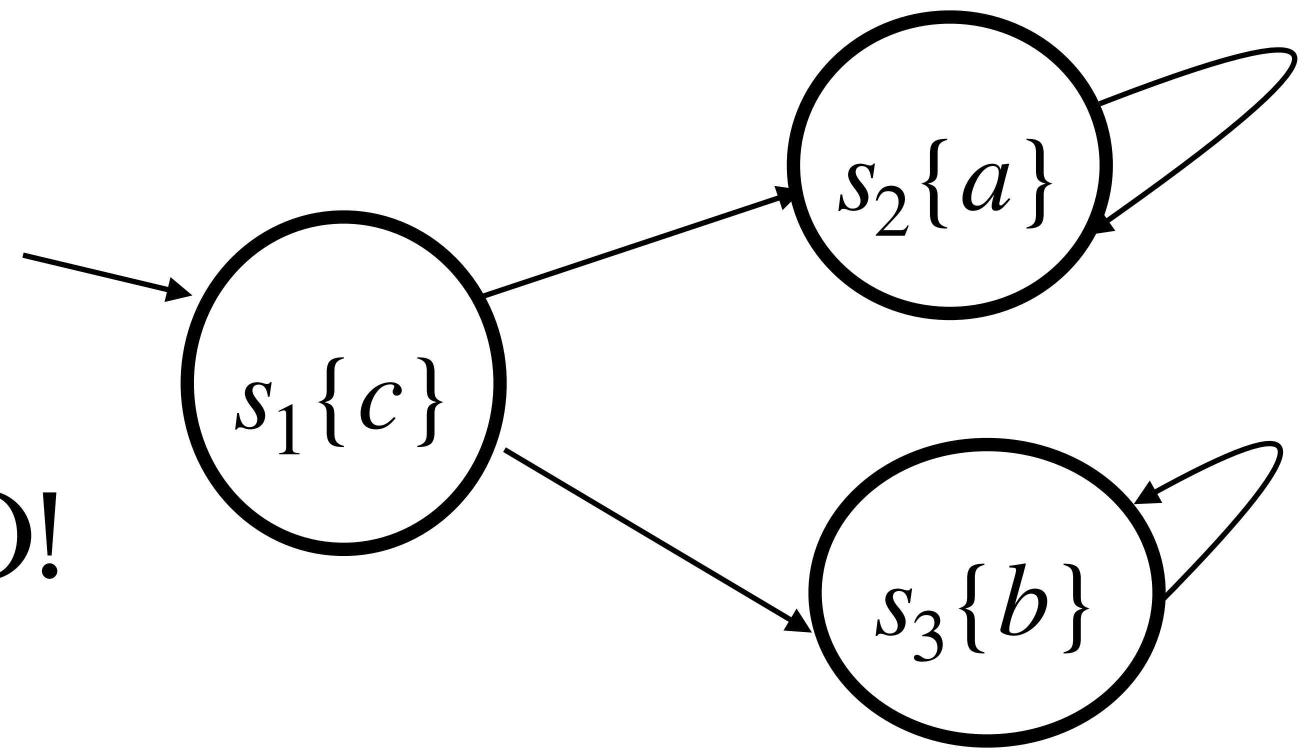

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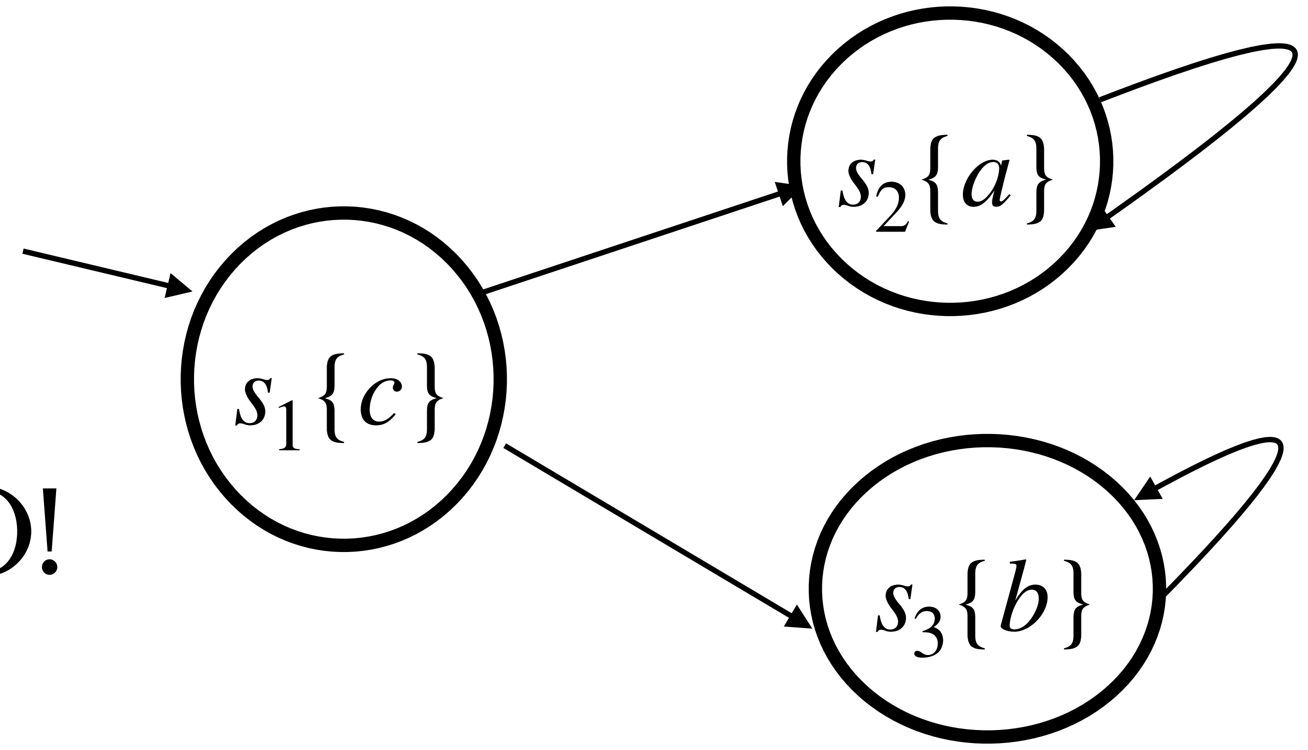

$$\forall \Diamond (a \land b) \equiv \forall \Diamond a \land \forall \Diamond b$$

CTL : Formula Equivalence

The formulae F_1, F_2 are said to be semantically equivalent if any state in any model that satisfies one also satisfies the other.

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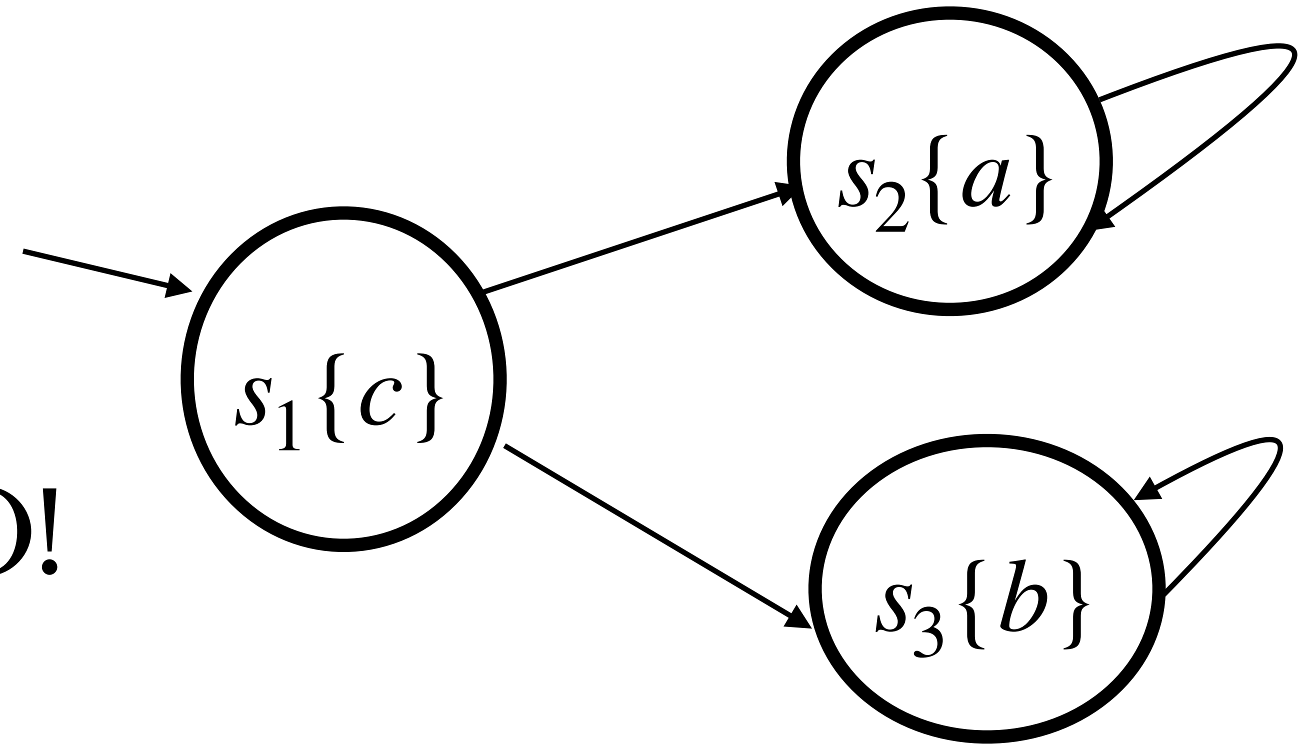
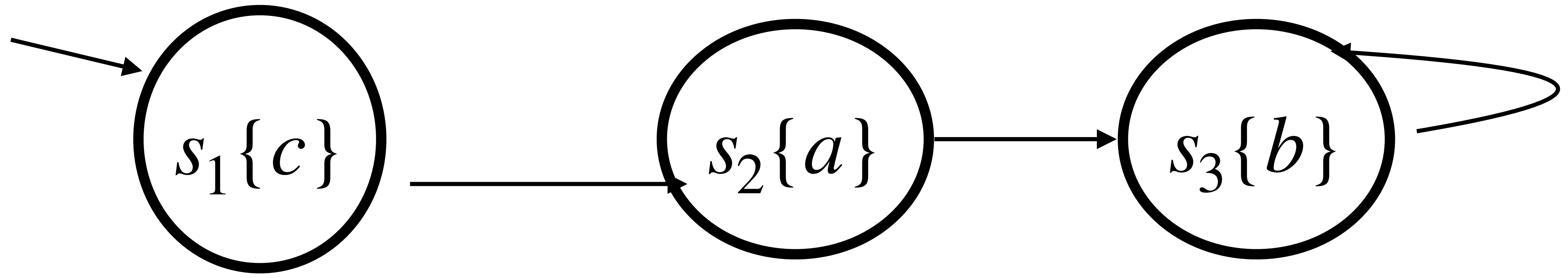


$$\forall \lozenge (a \wedge b) \equiv \forall \lozenge a \wedge \forall \lozenge b$$



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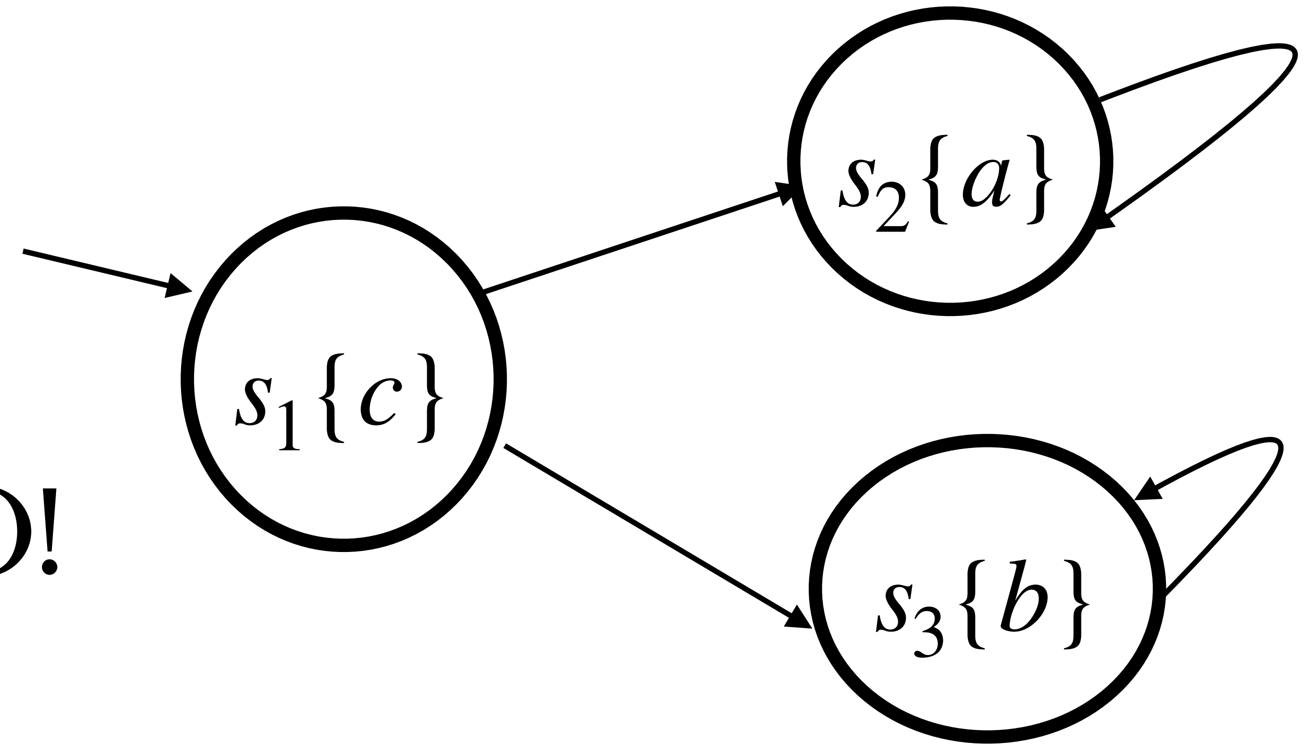

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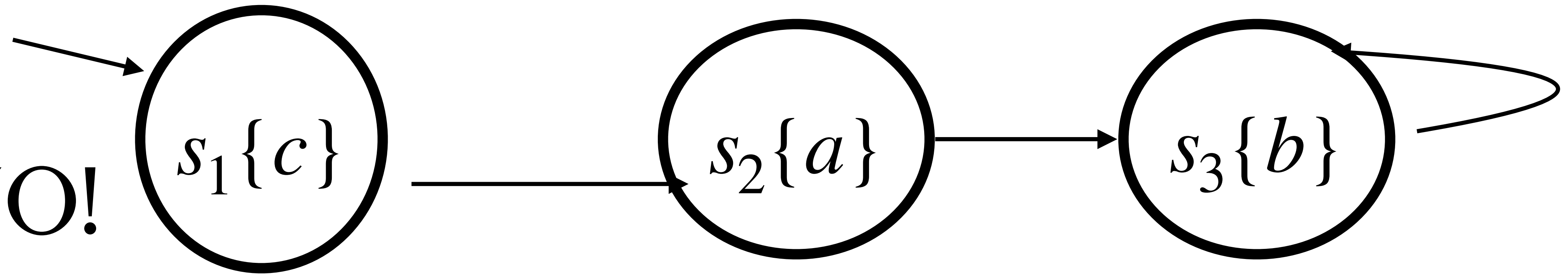

CTL : Formula Equivalence

$$\forall \Box (a \land b) \equiv \forall \Box a \land \forall \Box b$$

CTL : Formula Equivalence

$$\forall \Box (a \wedge b) \equiv \forall \Box a \wedge \forall \Box b$$

$$< M, s_i > \models \forall \Box (a \wedge b)$$

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$$\equiv \forall \pi \in \{s_o, s_1, s_2, \ldots, \} \quad \forall j \geq i, < M, s_j > \models (a \wedge b)$$

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$$\equiv \forall \pi \in \{s_o, s_1, s_2, \ldots, \} \quad \forall j \geq i, (< M, s_j > \models (a) \land < M, s_j > \models (b))$$

CTL : Formula Equivalence

$$\forall \square (a \wedge b) \equiv \forall \square a \wedge \forall \square b$$

$$< M, s_i > \vDash \forall \square (a \wedge b)$$

$$\equiv \forall \pi \in \{s_o, s_1, s_2, \ldots, \} \quad \forall j \geq i, < M, s_j > \vDash (a \wedge b)$$

$$\equiv \forall \pi \in \{s_o, s_1, s_2, \ldots, \} \quad \forall j \geq i, (< M, s_j > \vDash (a) \wedge < M, s_j > \vDash (b))$$

$$\equiv \forall \pi \in \{s_o, s_1, s_2, \ldots, \}$$
$$\forall j \geq i, < M, s_j > \vDash (a) \wedge \forall \pi \in \{s_o, s_1, s_2, \ldots, \} \forall j \geq i, < M, s_j > \vDash (b)$$

CTL : Formula Equivalence

$$\forall \Box (a \wedge b) \equiv \forall \Box a \wedge \forall \Box b$$

$$< M, s_i > \models \forall \Box (a \wedge b)$$

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$$\equiv < M, s_i > \models \forall \Box a \wedge \forall \Box b$$

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$$\equiv < M, s_i > \models \forall \Box a \wedge \forall \Box b$$

$$\forall \Box (a \wedge b) \equiv \forall \Box a \wedge \forall \Box b$$

CTL : Formula Equivalence

$$\exists \Diamond (a \vee b) \stackrel{?}{\equiv} \exists \Diamond a \vee \exists \Diamond b$$

$$\forall \mathbf{N} \forall \Box a \stackrel{?}{\equiv} \forall \Box \forall \mathbf{N} a$$

$$\exists \mathbf{N} \exists \Box a \stackrel{?}{\equiv} \exists \Box \exists \mathbf{N} a$$

CTL : Weak Until

How to write Until in terms of equivalent weak until?

$$F_1 \mathbf{U} F_2 \equiv$$

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$$F_1 \mathbf{U} F_2 \equiv (F_1 \mathbf{W} F_2) \land \Diamond F_2$$

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How to write Until in terms of equivalent weak until?

$$F_1 \mathbf{U} F_2 \equiv (F_1 \mathbf{W} F_2) \wedge \Diamond F_2$$

$$F_1 \mathbf{W} F_2 \equiv (F_1 \mathbf{U} F_2) \vee \Box F_1$$

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$$\neg(F_1 \mathbf{U} F_2) \equiv$$

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$$\neg(F_1 \mathbf{U} F_2) \equiv (F_1 \wedge \neg F_2) \mathbf{U} (\neg F_1 \wedge \neg F_2) \vee \Box (F_1 \wedge \neg F_2)$$

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$$\equiv (F_1 \wedge \neg F_2) \mathbf{U} (\neg F_1 \wedge \neg F_2)$$

CTL : Weak Until

$$\neg(F_1 \mathbf{W} F_2) \equiv (F_1 \wedge \neg F_2) \mathbf{U} (\neg F_1 \wedge \neg F_2)$$

$$\forall(F_1 \mathbf{W} F_2) \equiv$$

$$\exists(F_1 \mathbf{W} F_2) \equiv$$

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$$\exists(F_1 \mathbf{W} F_2) \equiv$$

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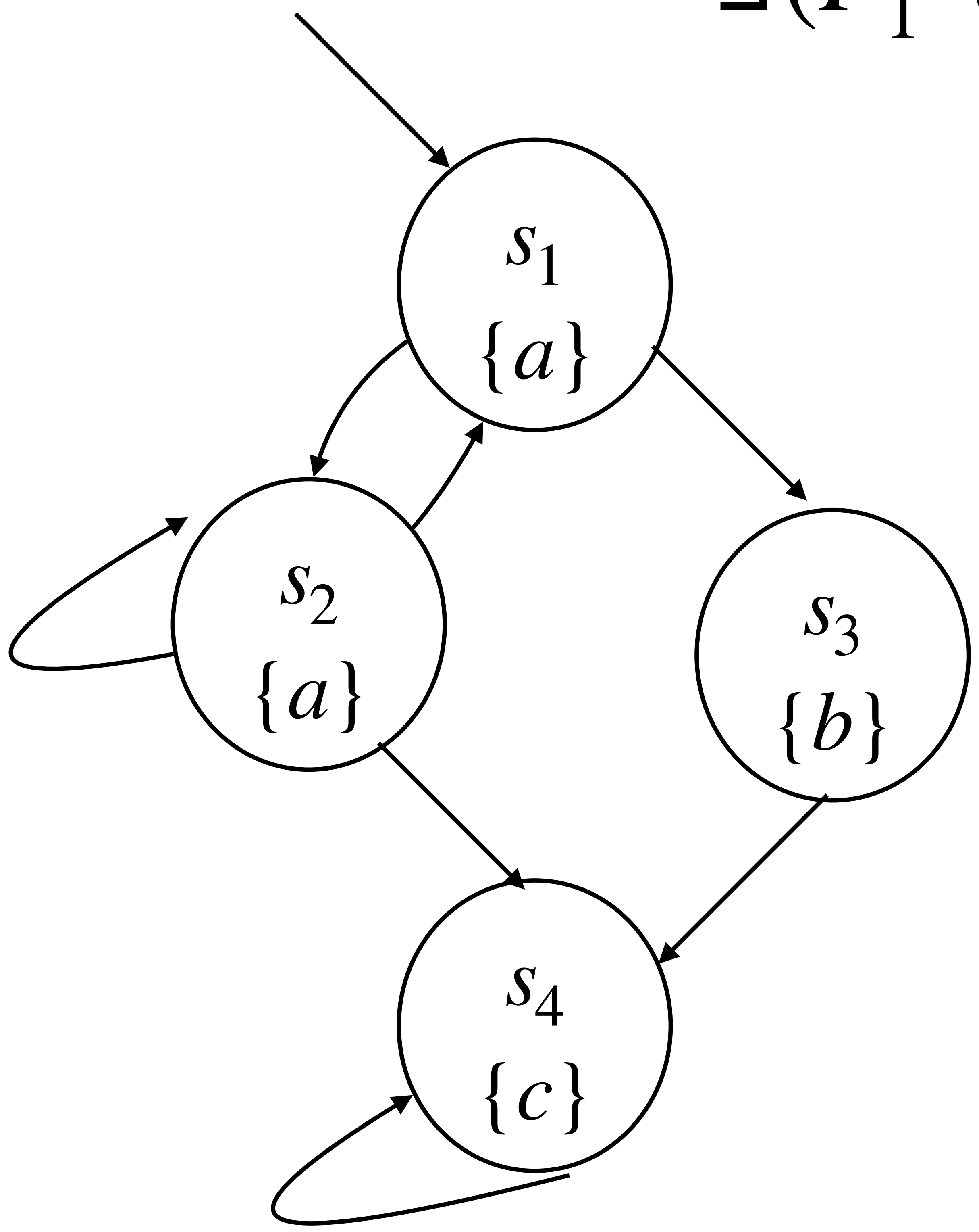

CTL : Weak Until

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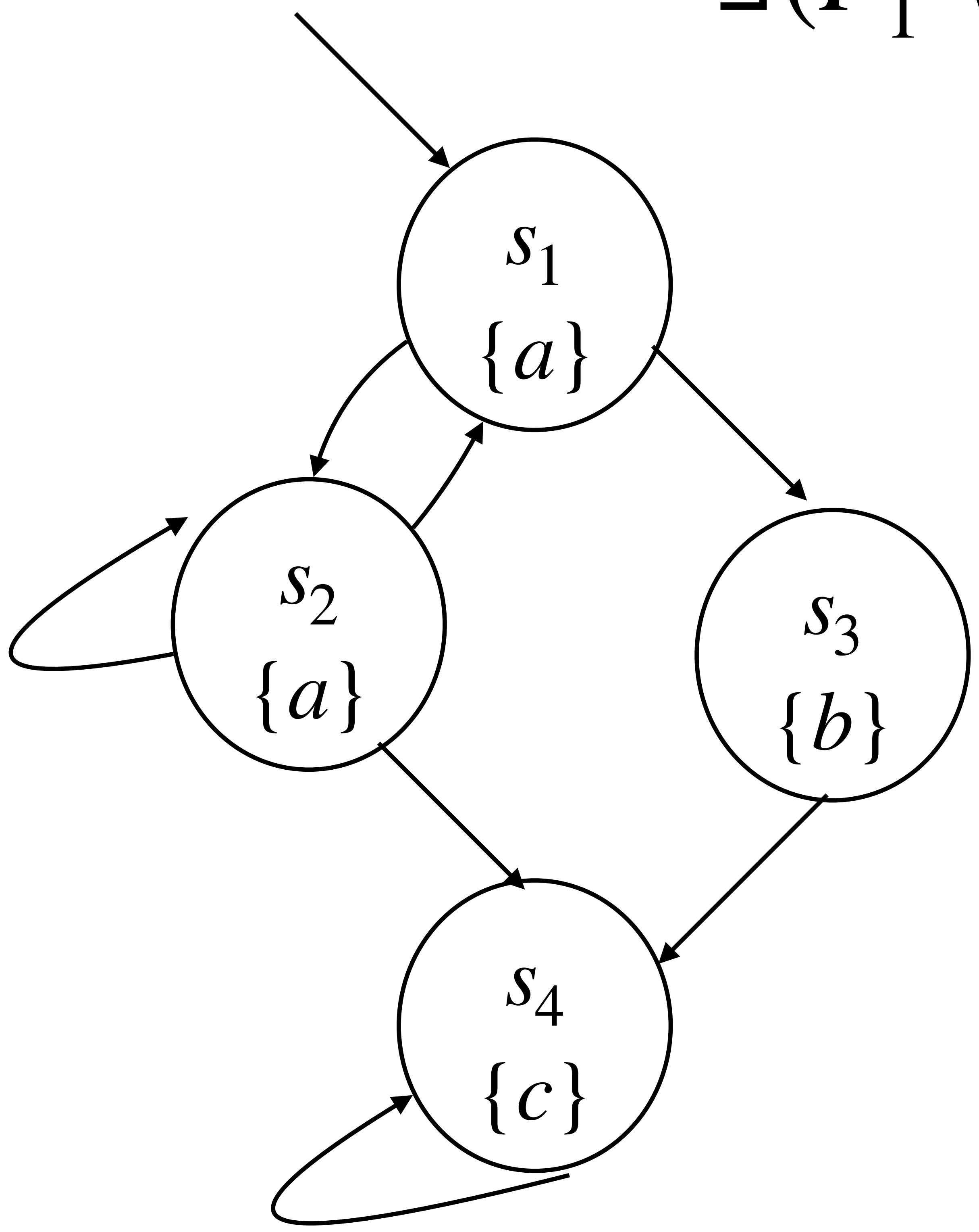

$$M \overset{?}{\models} \forall \Diamond \exists (a \mathbf{W} c)$$

CTL : Weak Until

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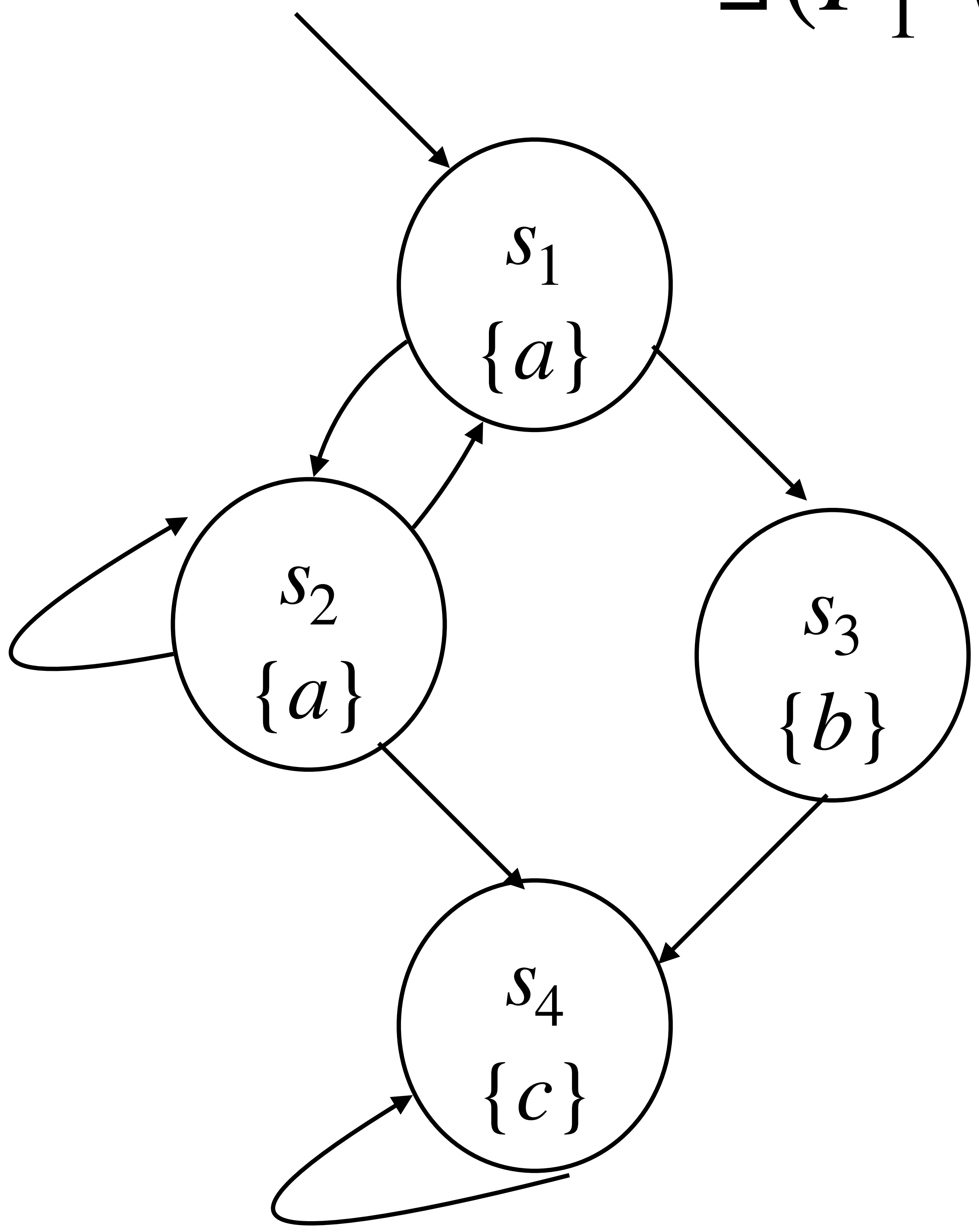

$$M \overset{?}{\models} \forall \Diamond \exists(a \mathbf{W} c)$$ YES

CTL : Weak Until

$$\neg(F_1 \ \mathbf{W} \ F_2) \equiv (F_1 \land \neg F_2) \ \mathbf{U} \ (\neg F_1 \land \neg F_2)$$

$$\forall(F_1 \ \mathbf{W} \ F_2) \equiv \neg \exists(F_1 \land \neg F_2) \ \mathbf{U} \ (\neg F_1 \land \neg F_2)$$

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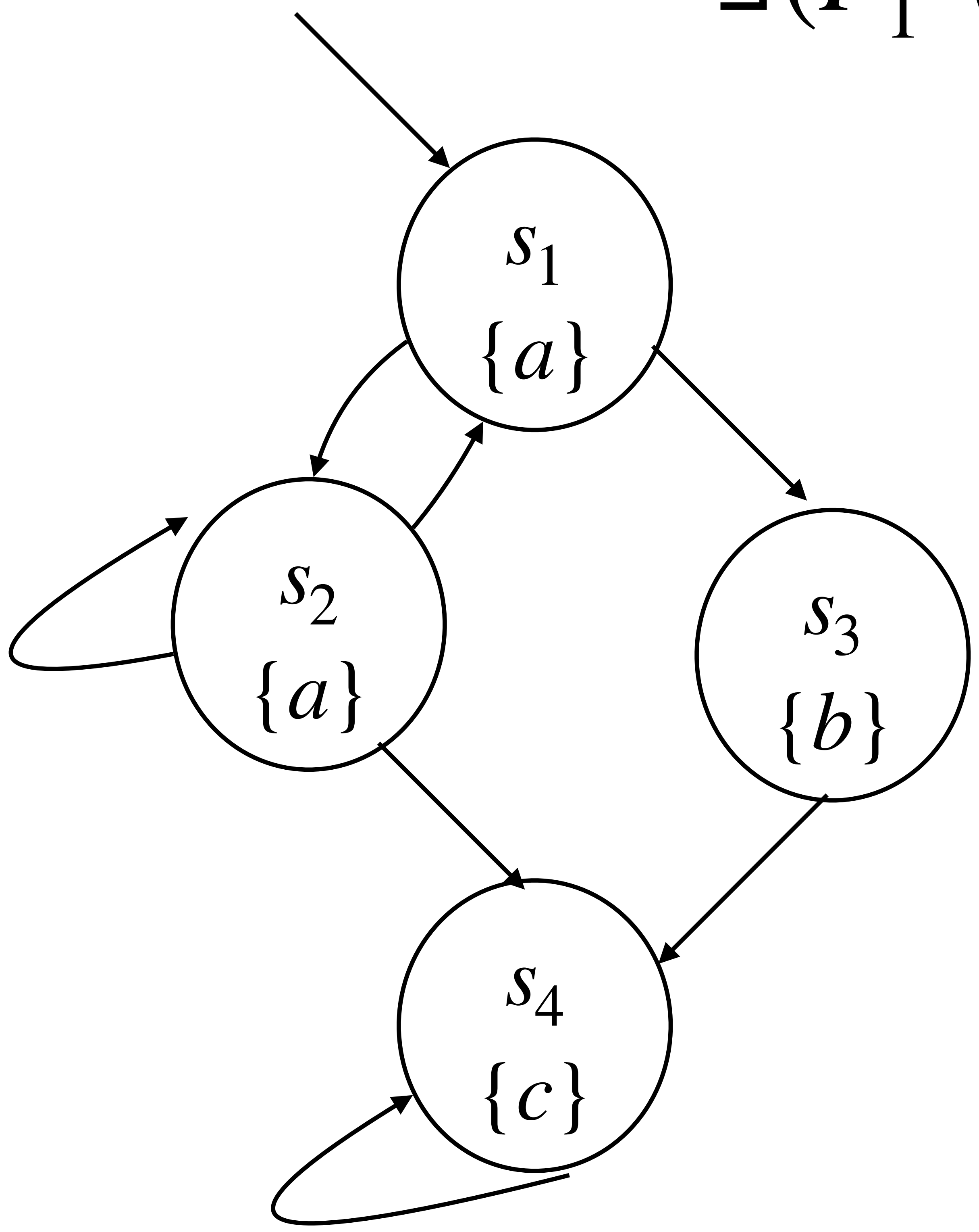

$$M \overset{?}{\models} \forall \Diamond \exists(a \mathbf{W} c) \quad \text{YES}$$

$$M \overset{?}{\models} \exists(a \mathbf{W} \ \exists \Diamond b)$$

CTL : Weak Until

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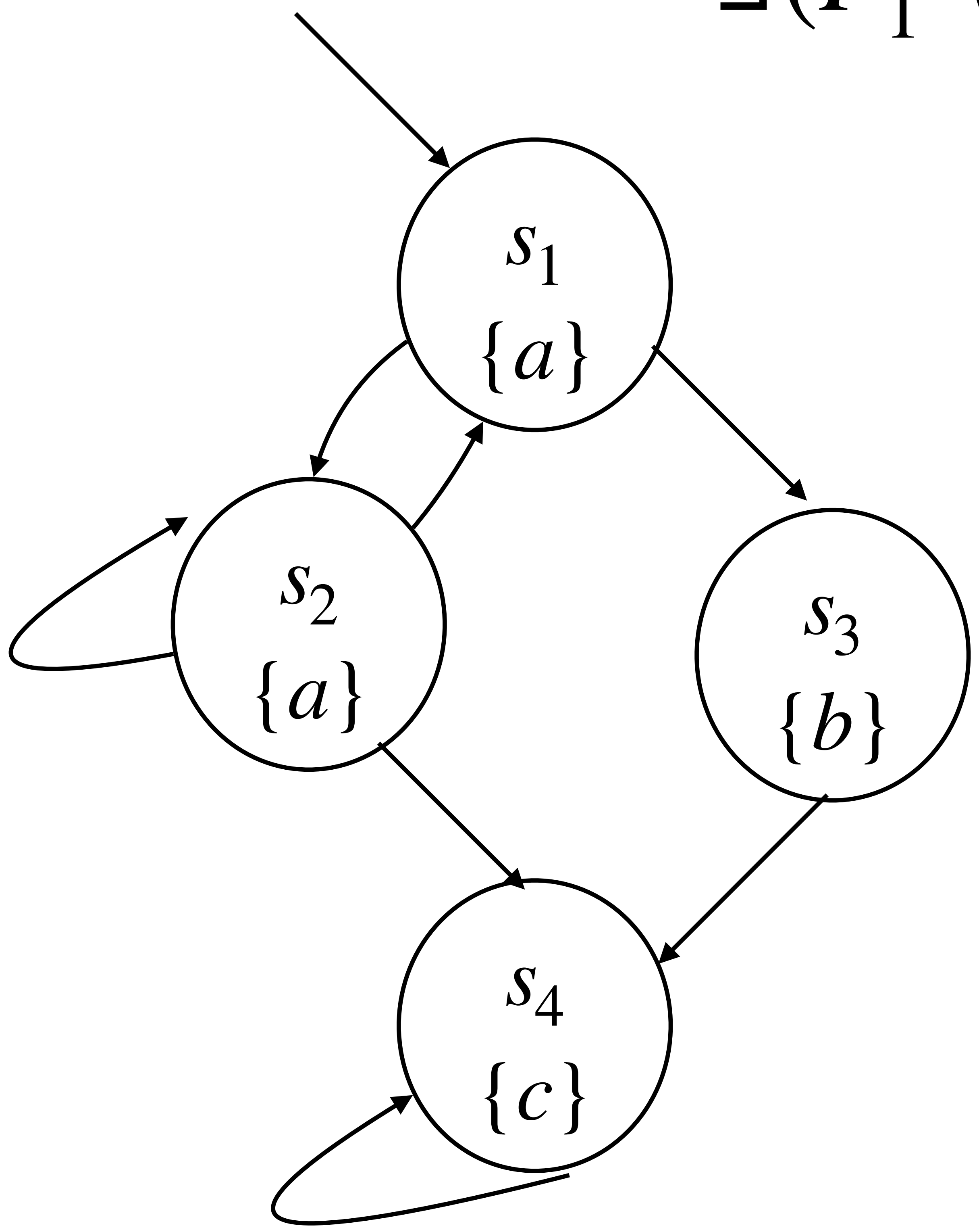

$M \overset{?}{\models} \forall \Diamond \exists (a \mathbf{W} c)$ YES

$M \overset{?}{\models} \exists (a \mathbf{W} \exists \Diamond b)$ YES

CTL : Weak Until

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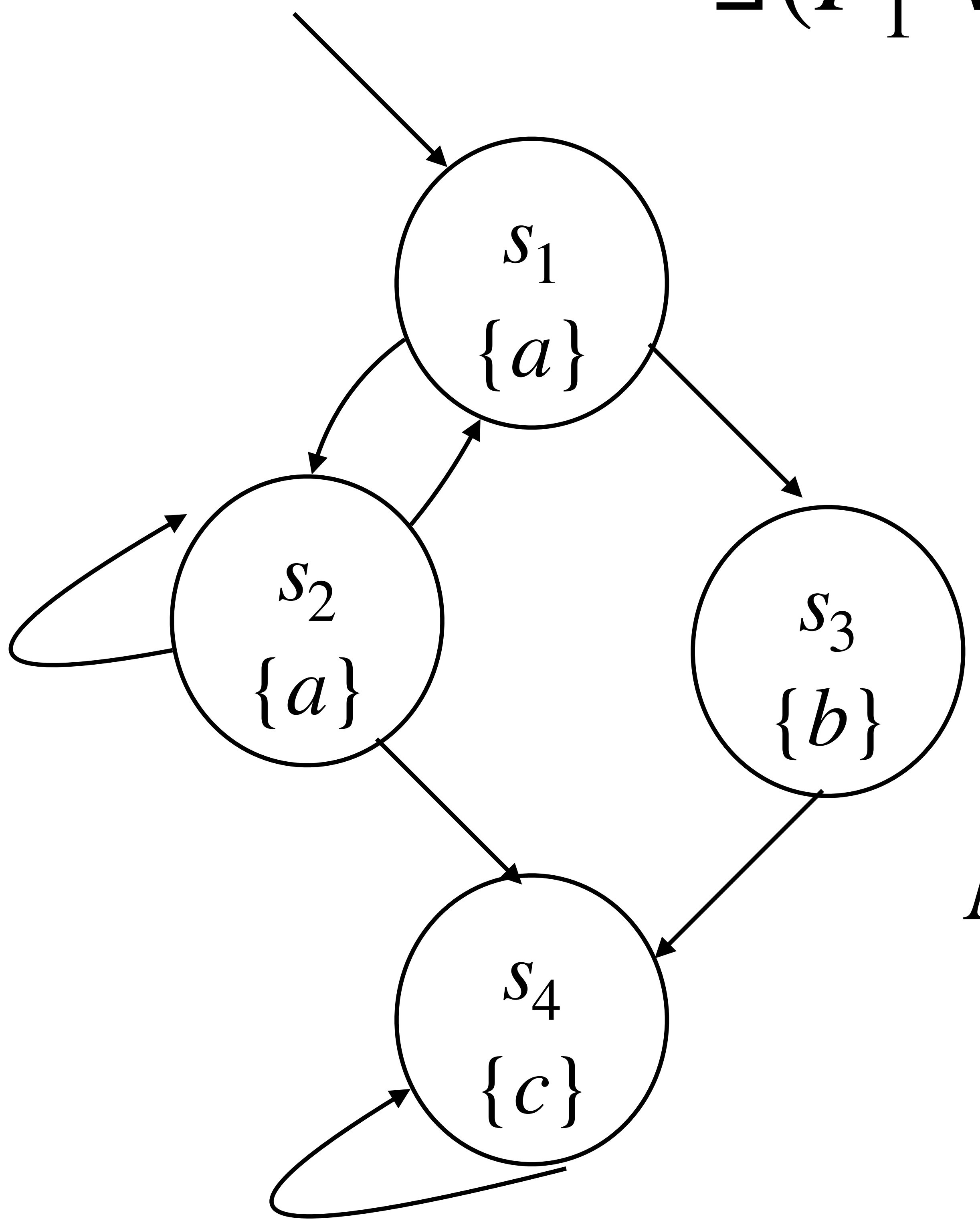

$M \overset{?}{\models} \forall \Diamond \exists(a\mathbf{W}c)$ YES

$M \overset{?}{\models} \exists(a\mathbf{W} \exists \Diamond b)$ YES

$M \overset{?}{\models} \forall((\exists \mathbf{N}(b \vee c))\mathbf{W} (a \wedge b))$

CTL : Weak Until

$$\neg(F_1 \mathbf{W} F_2) \equiv (F_1 \wedge \neg F_2) \mathbf{U} (\neg F_1 \wedge \neg F_2)$$

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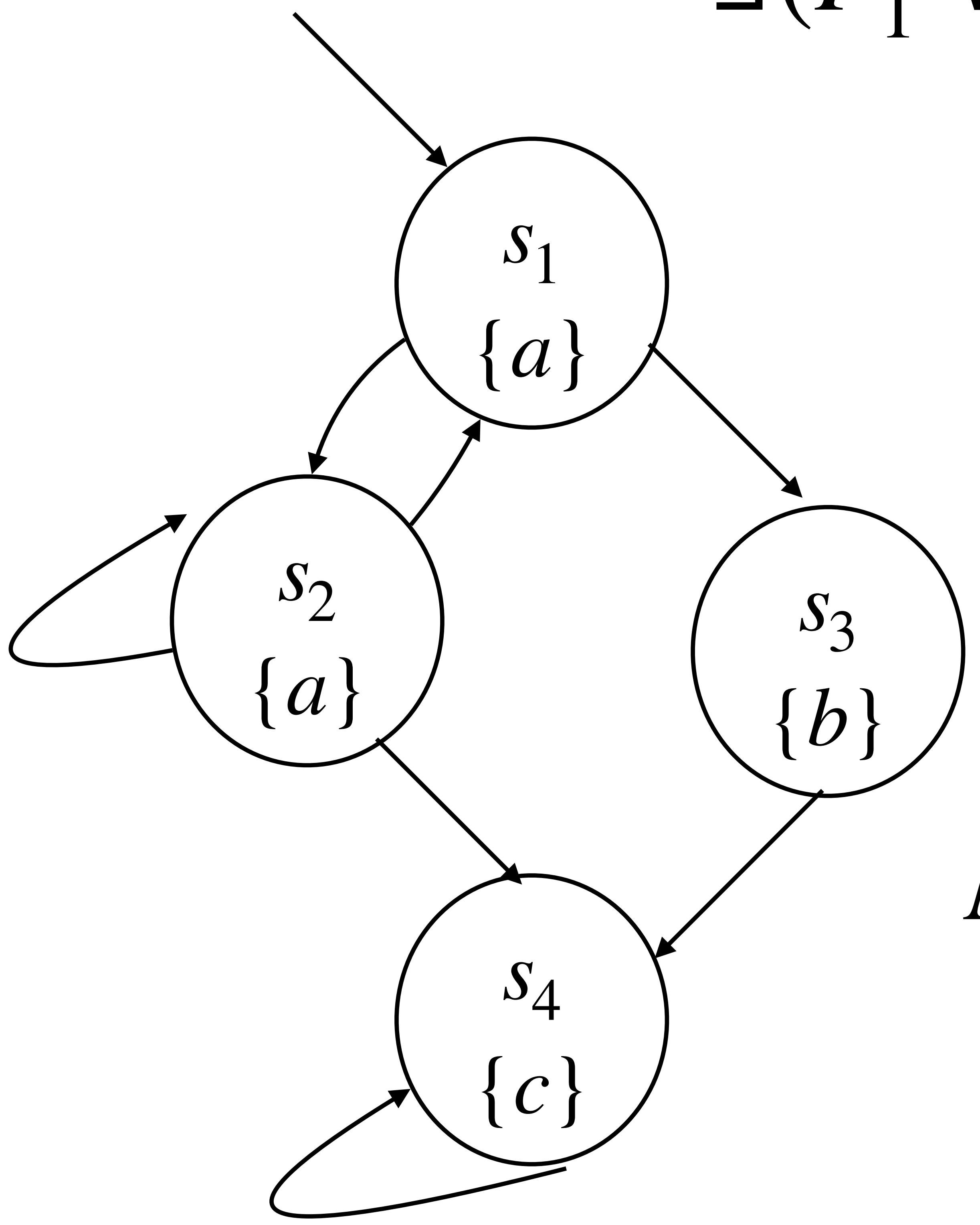
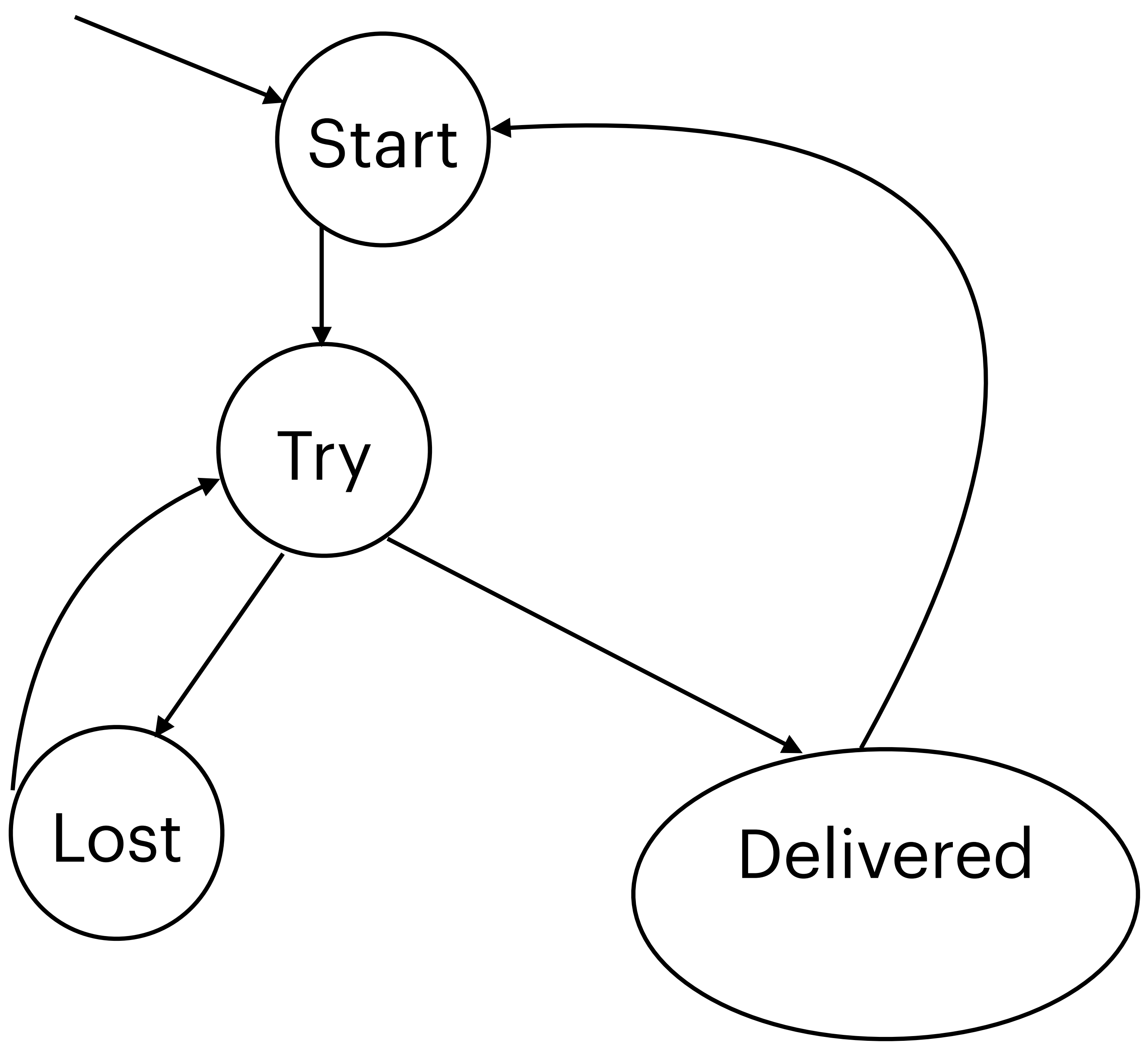


$M \overset{?}{\models} \forall \Diamond \exists (a \mathbf{W} c)$ YES

$M \overset{?}{\models} \exists (a \mathbf{W} \exists \Diamond b)$ YES

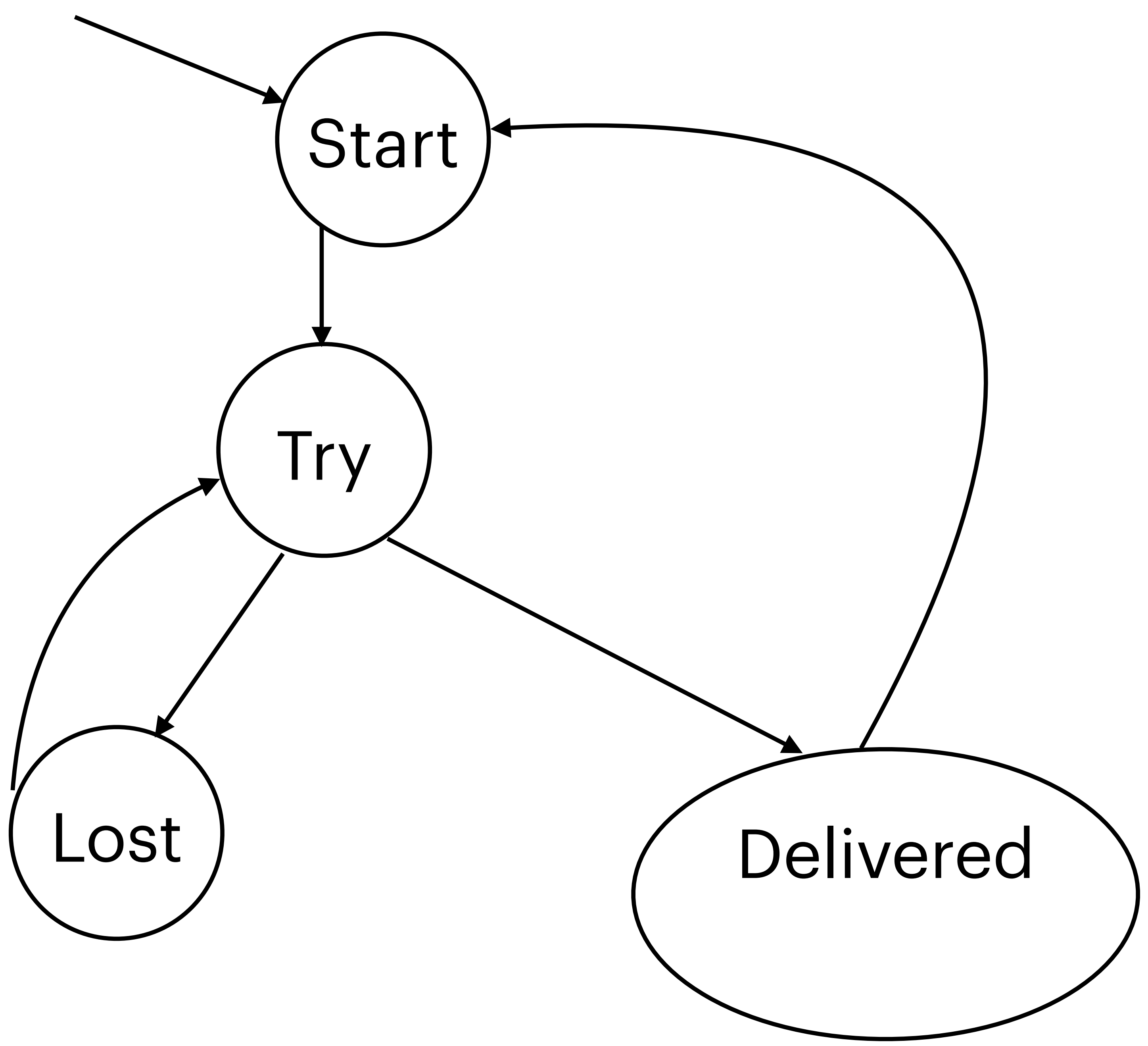
$M \overset{?}{\models} \forall ((\exists \mathbf{N}(b \vee c)) \mathbf{W} (a \wedge b))$ YES

CTL : Example



$$M \overset{?}{\models} \forall \Box \forall \Diamond start$$

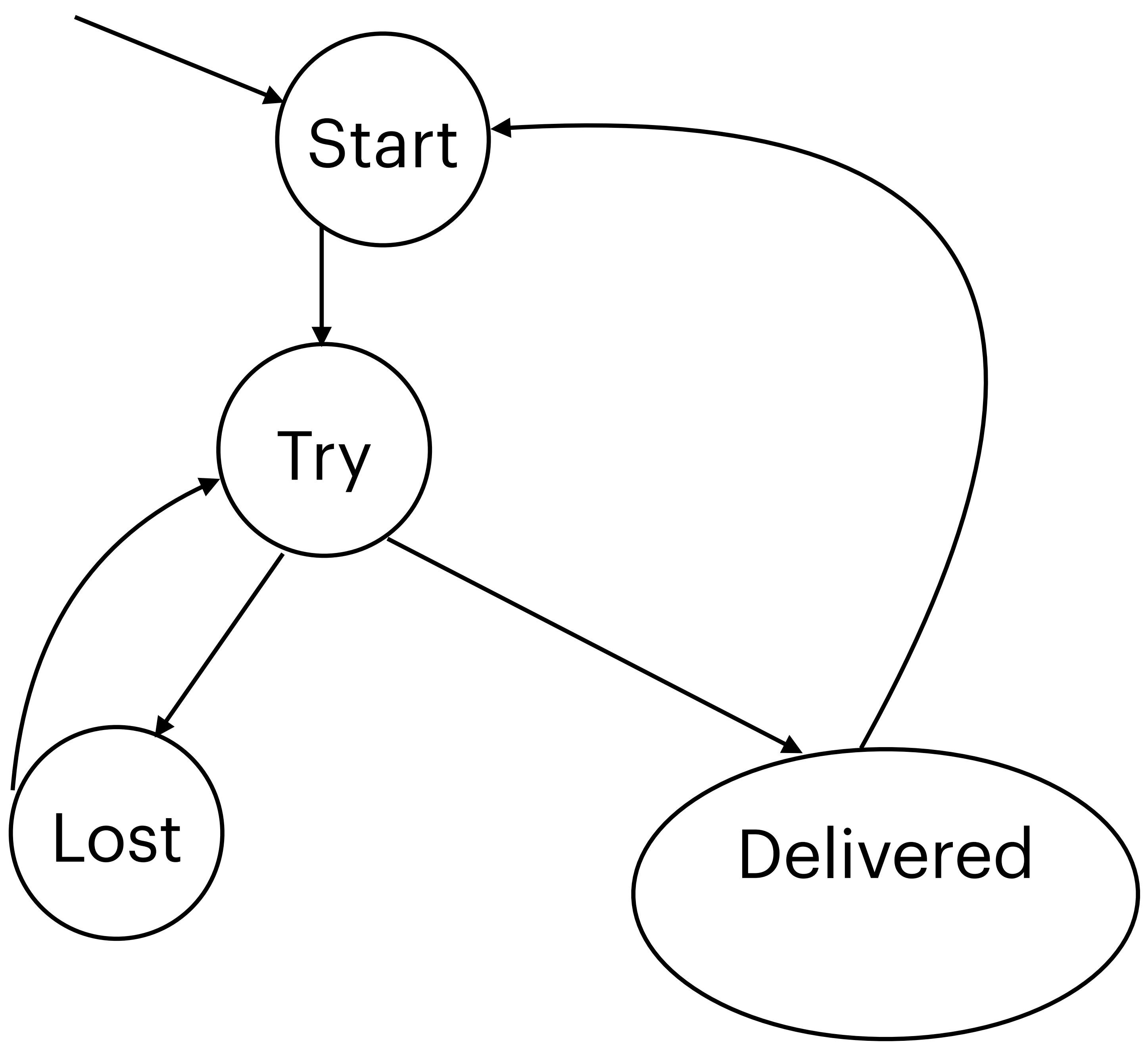
CTL : Example



$$M \overset{?}{\models} \forall \square \forall \lozenge start$$ No!

"Infinitely often start"

CTL : Example

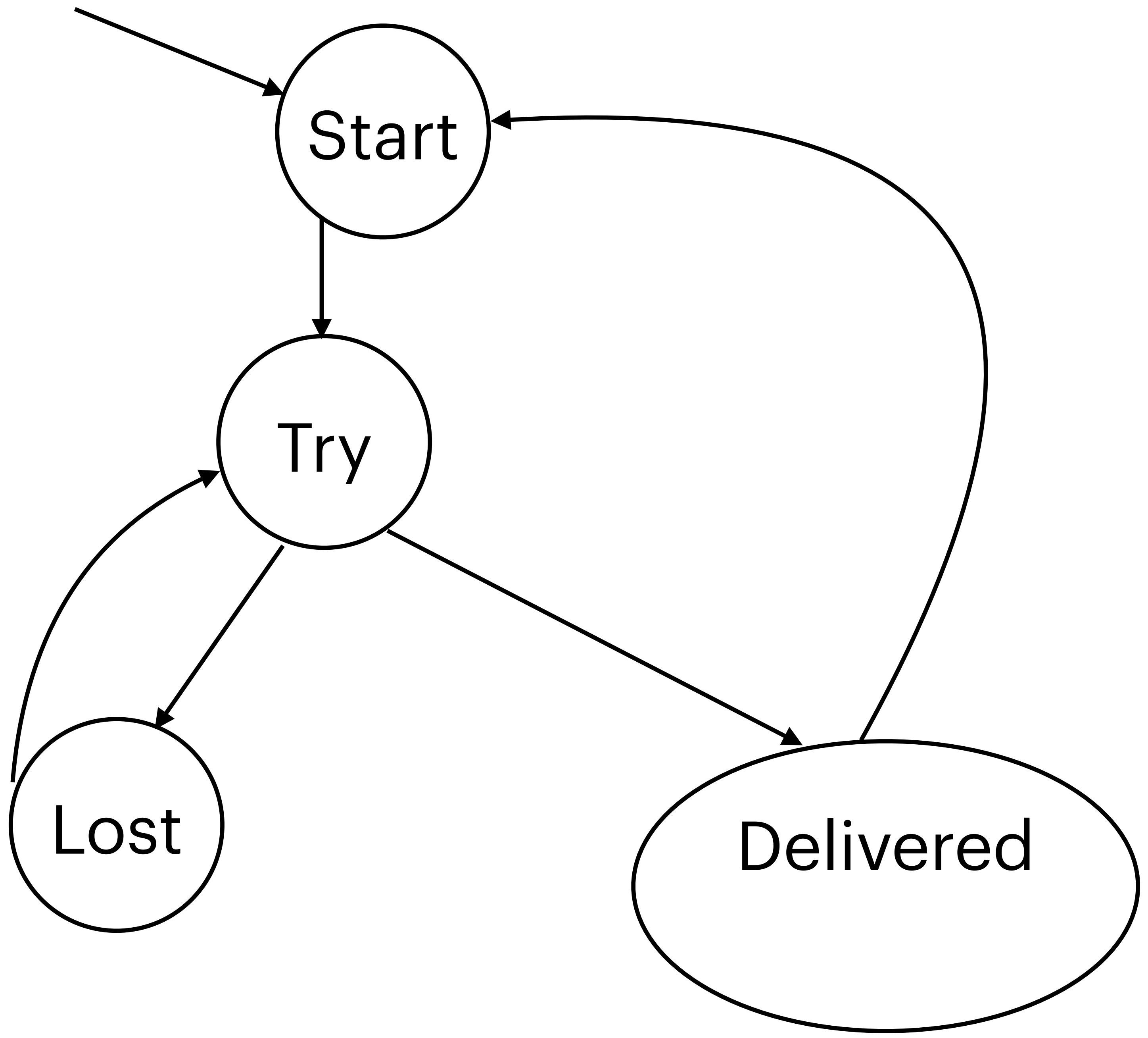


$M \overset{?}{\models} \forall \Box \forall \Diamond start$ No!

"Infinitely often start"

$M \overset{?}{\models} \exists \Diamond \forall \Box \neg start$

CTL : Example



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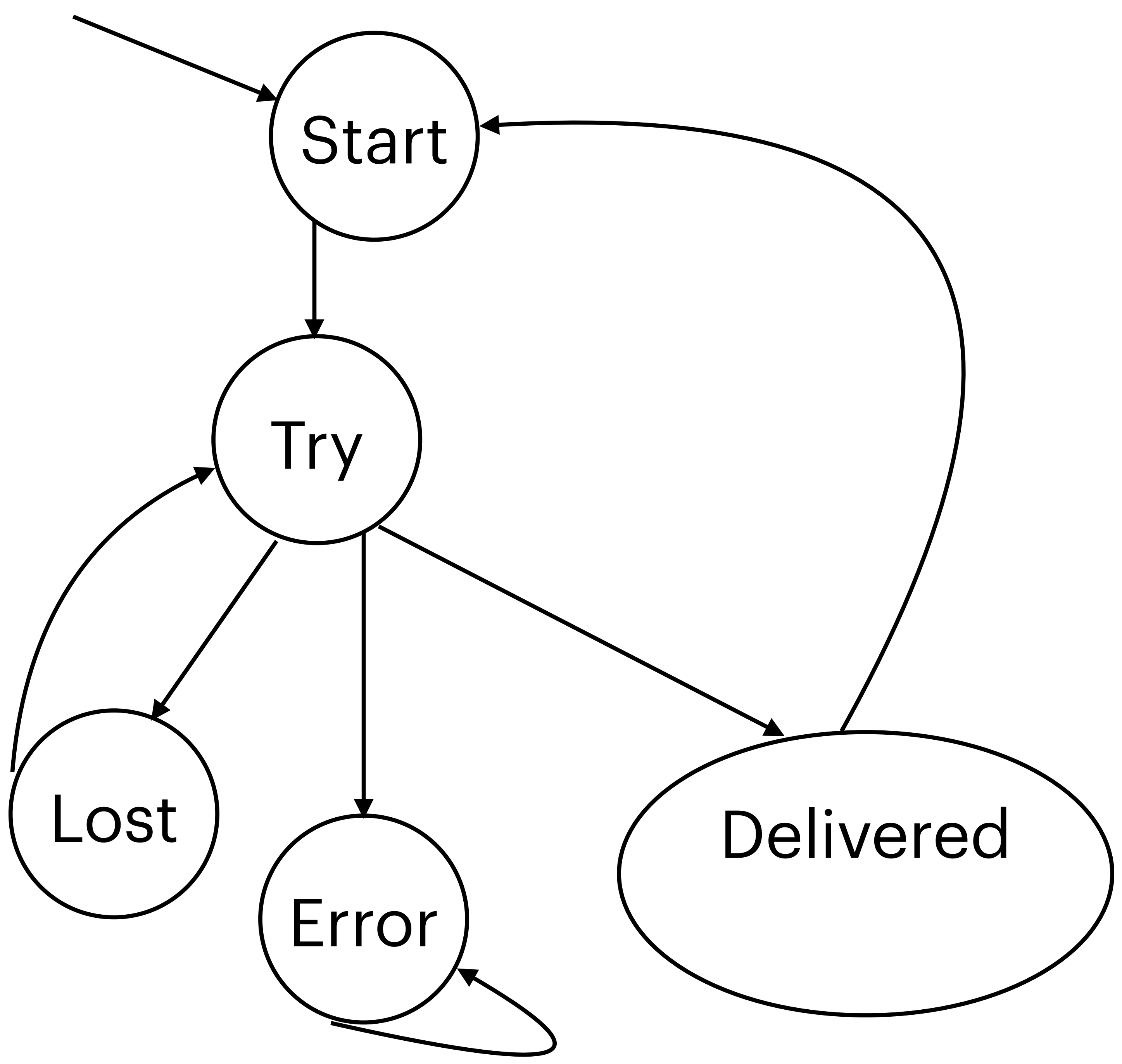

$M \overset{?}{\models} \forall \Box \forall \Diamond start$ No!

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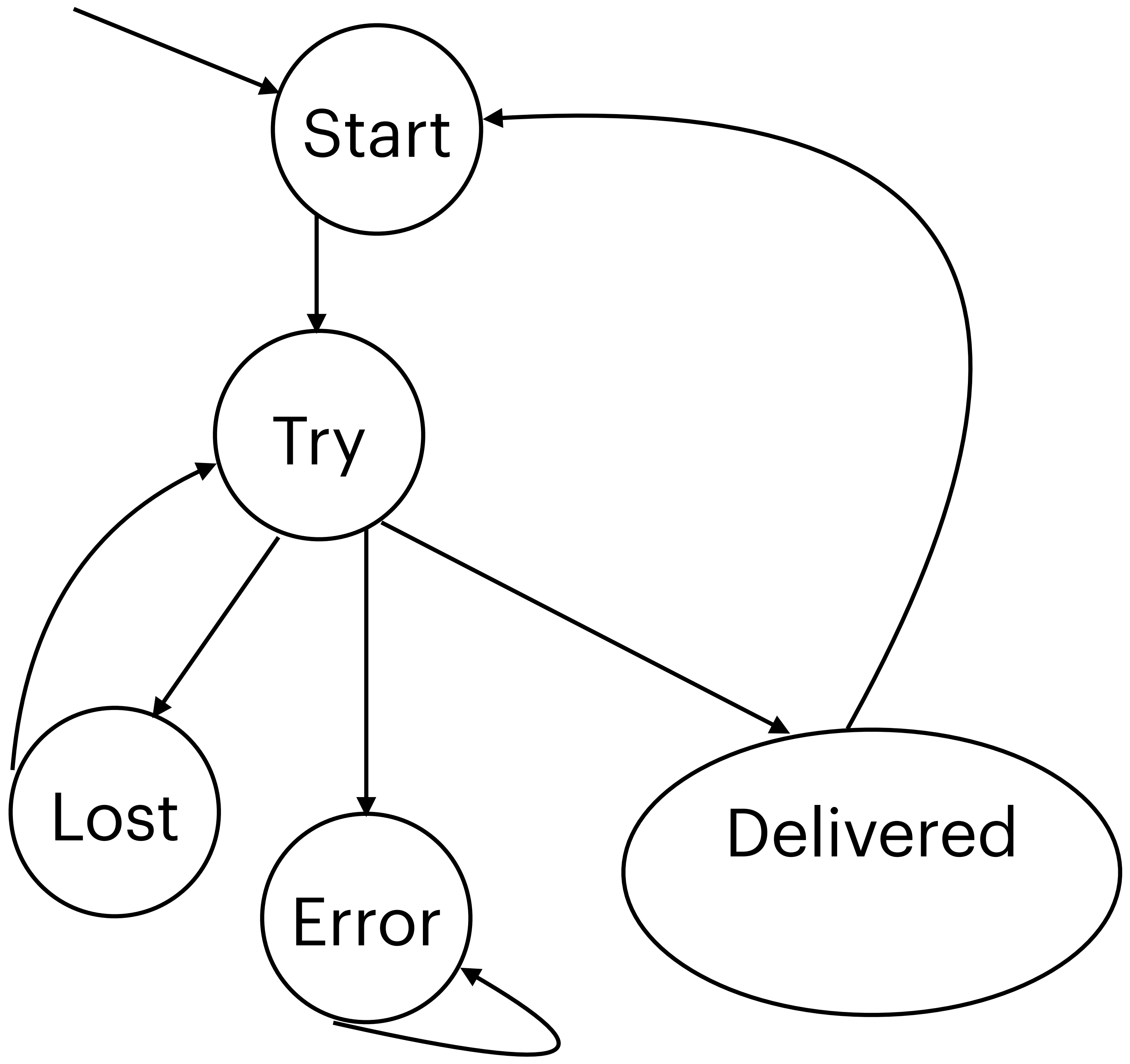


$M \overset{?}{\models} \forall \Box \forall \Diamond start$ No!

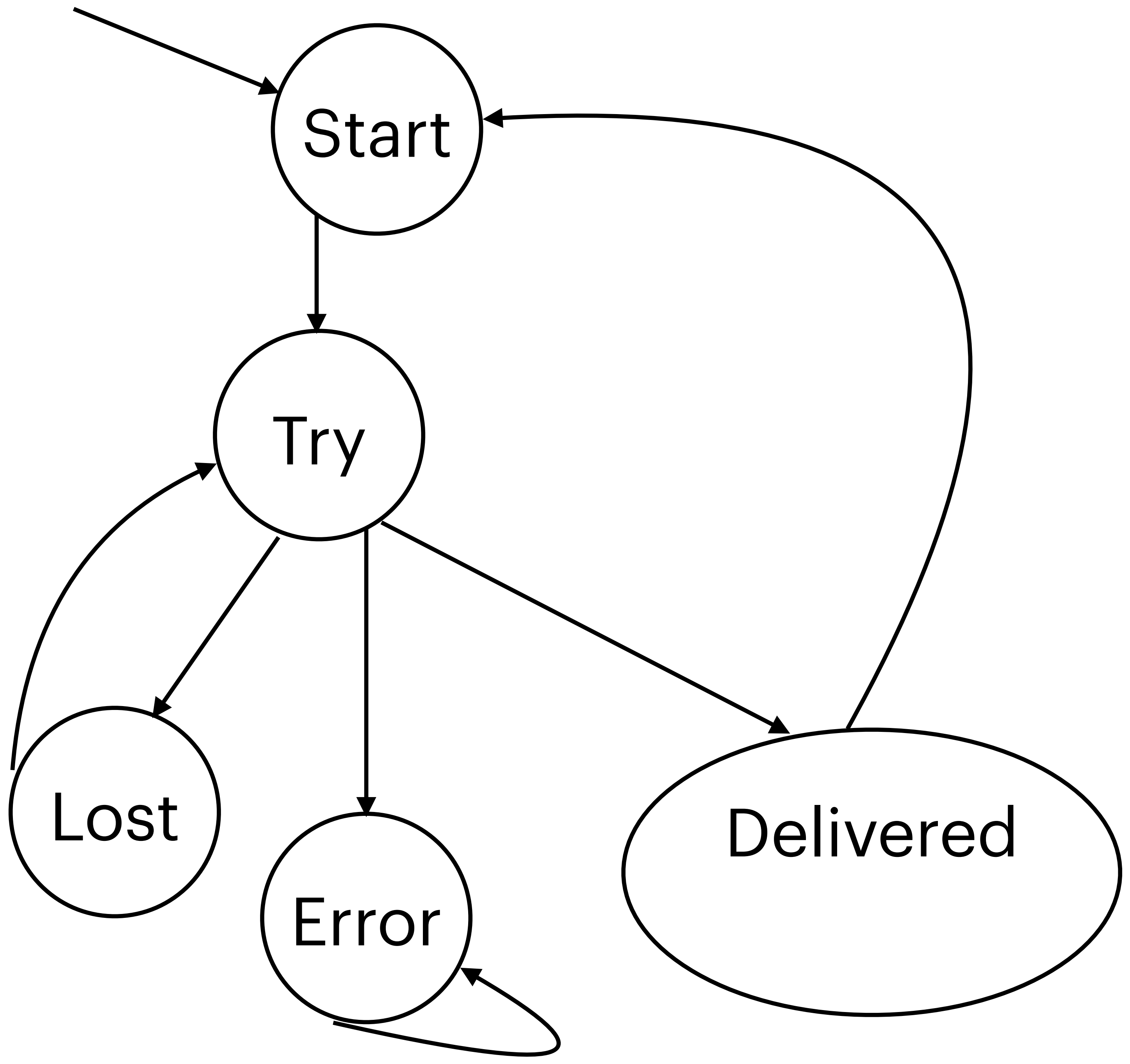
"Infinitely often start"

$M \overset{?}{\models} \exists \Diamond \forall \Box \neg start$ No!

After introducing "error" state.

$M \overset{?}{\models} \exists \Diamond \forall \Box \neg start$

CTL : Example



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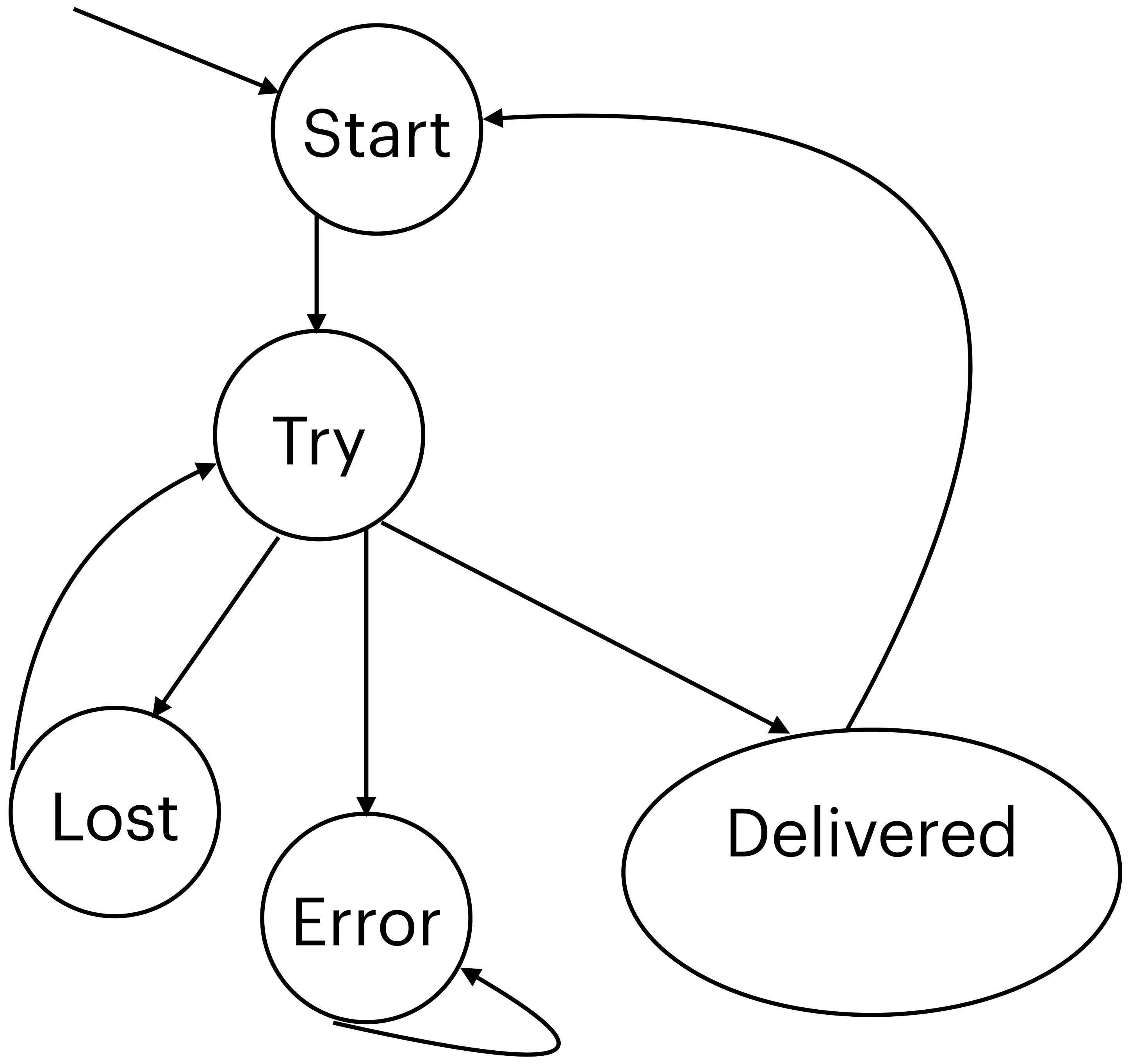
After introducing "error" state.

$M \overset{?}{\models} \exists \Diamond \forall \Box \neg start$ Yes!

$M \overset{?}{\models} \forall \mathbf{N} \exists \mathbf{N} \forall \Box \neg start$

CTL : Example



$M \overset{?}{\models} \forall \Box \forall \Diamond start$ No!

"Infinitely often start"

$M \overset{?}{\models} \exists \Diamond \forall \Box \neg start$ No!

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$M \overset{?}{\models} \exists \Diamond \forall \Box \neg start$ Yes!

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CTL :Examples

Correlation: $\Diamond p \rightarrow \Diamond q$ What will be the equivalent CTL formula?

$\forall \Diamond p \rightarrow \forall \Diamond q$ If all the paths have p along them then all the paths have q along them!

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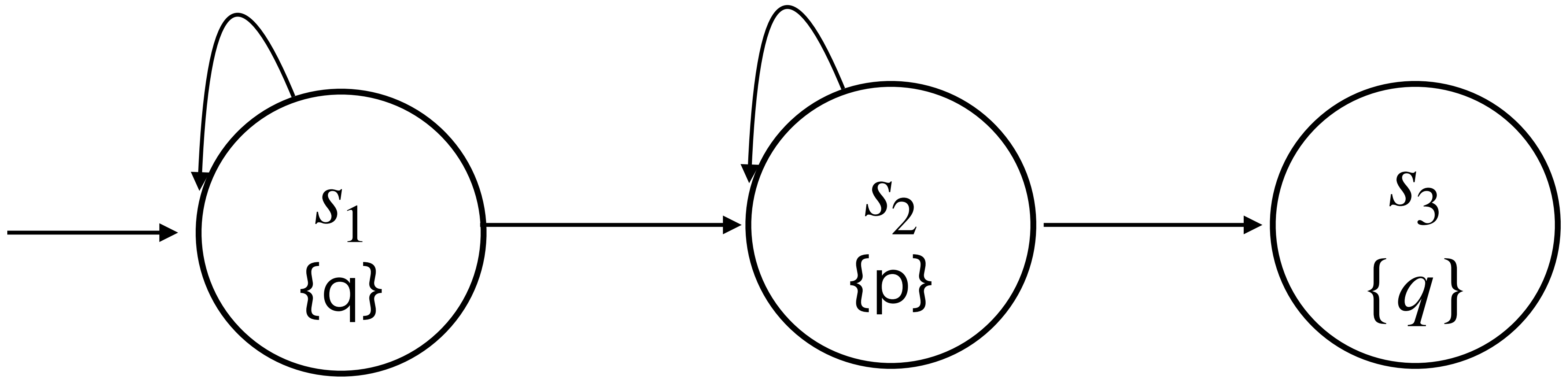

$\pi_1 = q, q, q, q, q, \ldots$ $\pi_2 = q, q, q, p, p, \ldots$ $\pi_3 = q, q, q, \ldots, p, p, \ldots, q$

$< M, \pi_2 > \nvDash F, < M, s_1 > \nvDash F, < M > \nvDash F$

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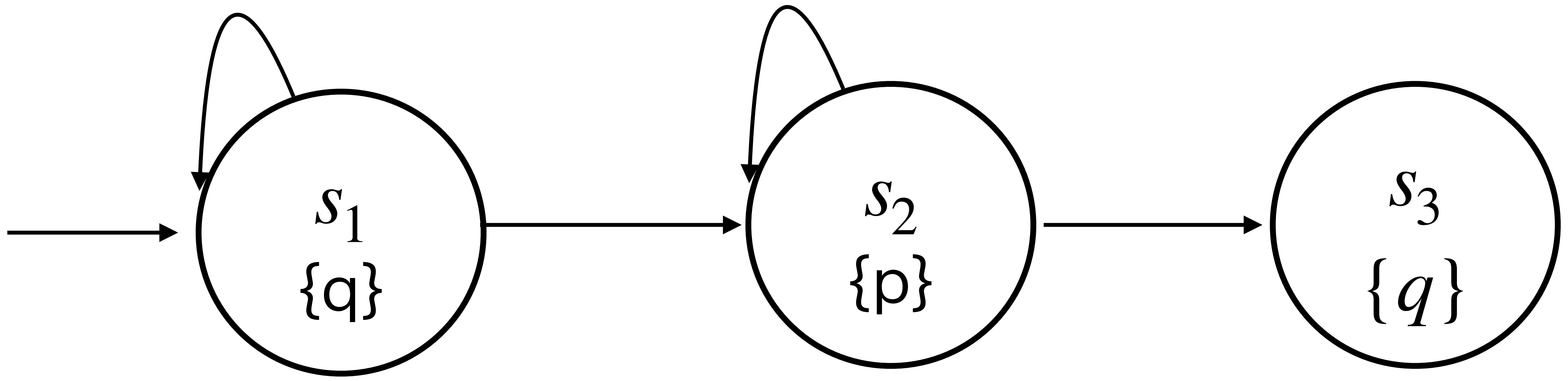

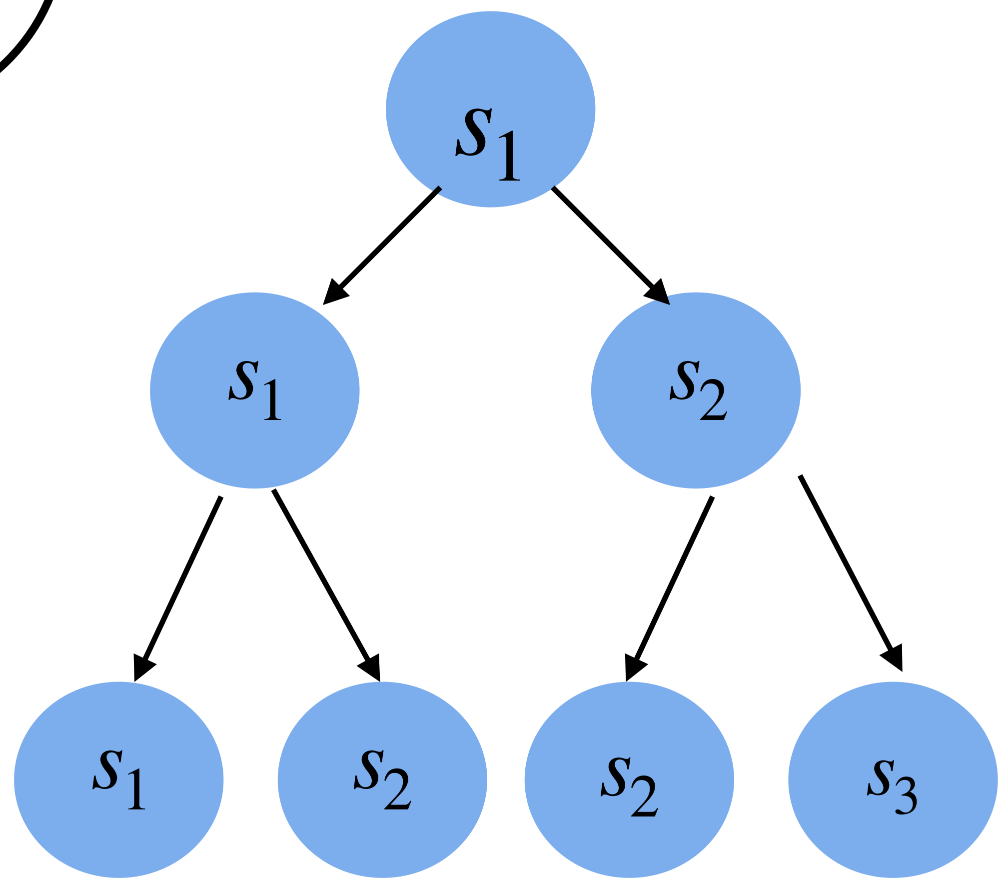

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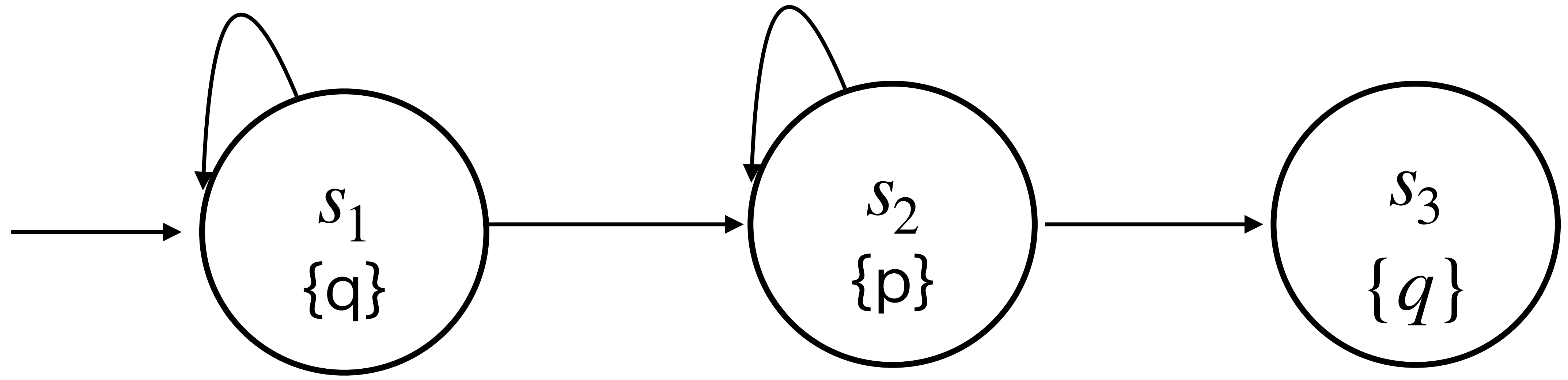

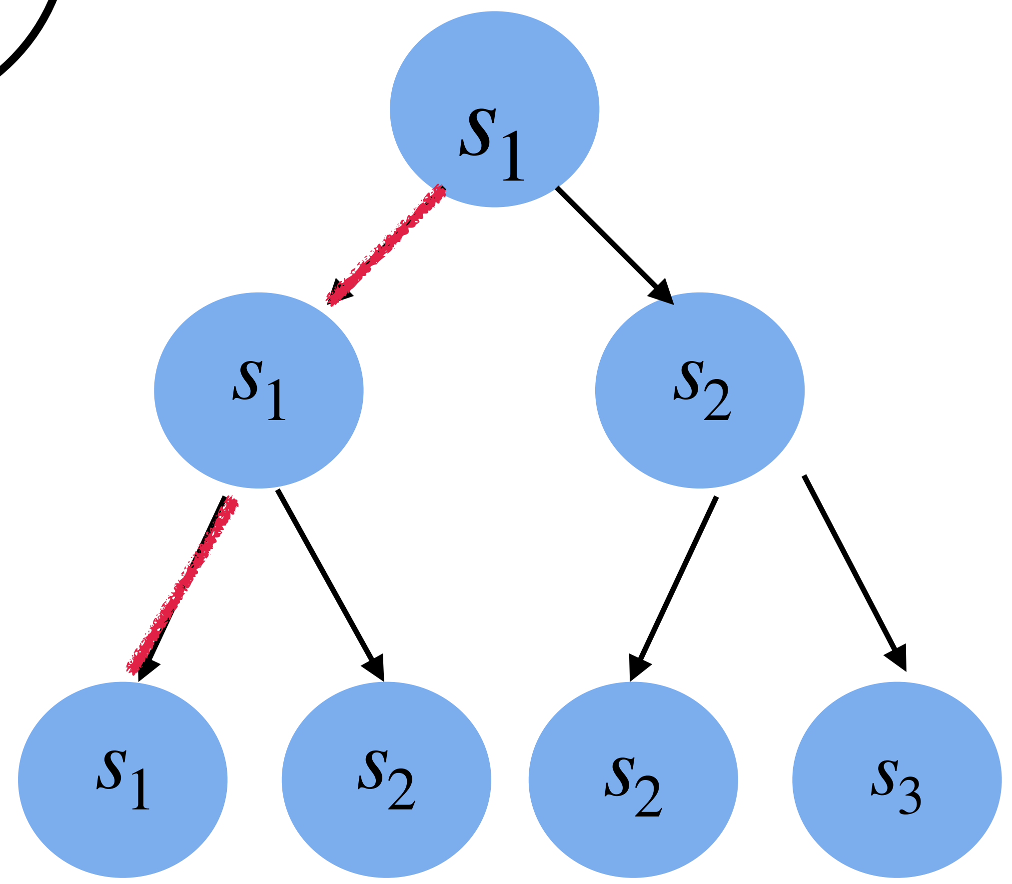

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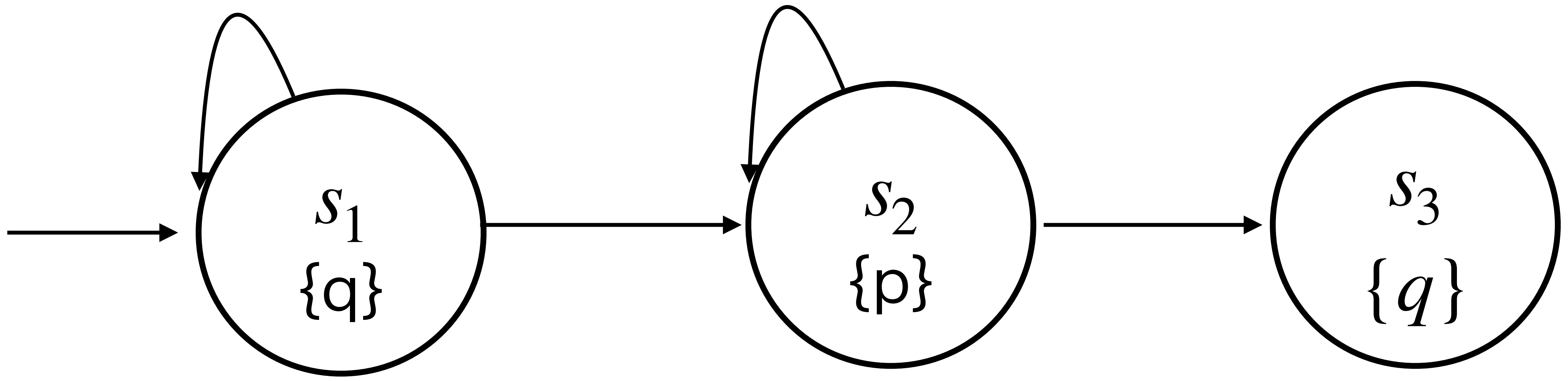

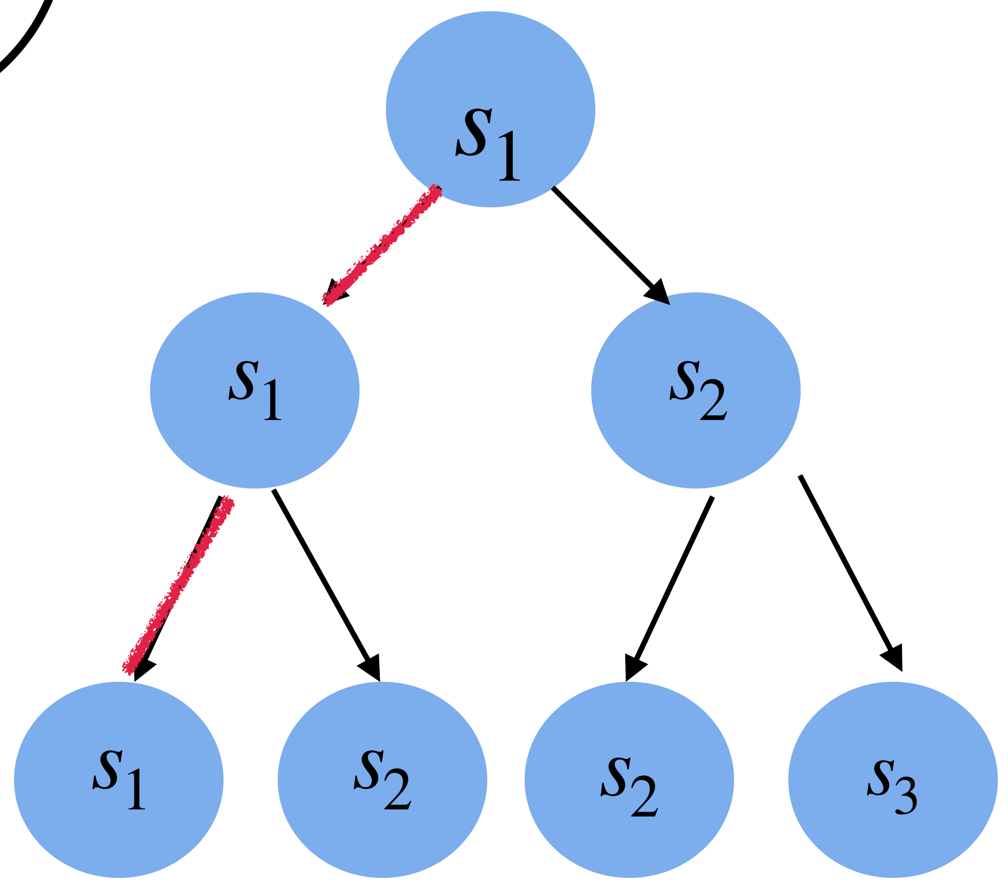

$\forall \Diamond p$ is False, hence F_1 is trivially True!

$$< M > \vDash F_1$$

CTL :Examples

Correlation: $\Diamond p \rightarrow \Diamond q$ What will be the equivalent CTL formula?

$$\forall \Box (p \rightarrow \forall \Diamond q)$$

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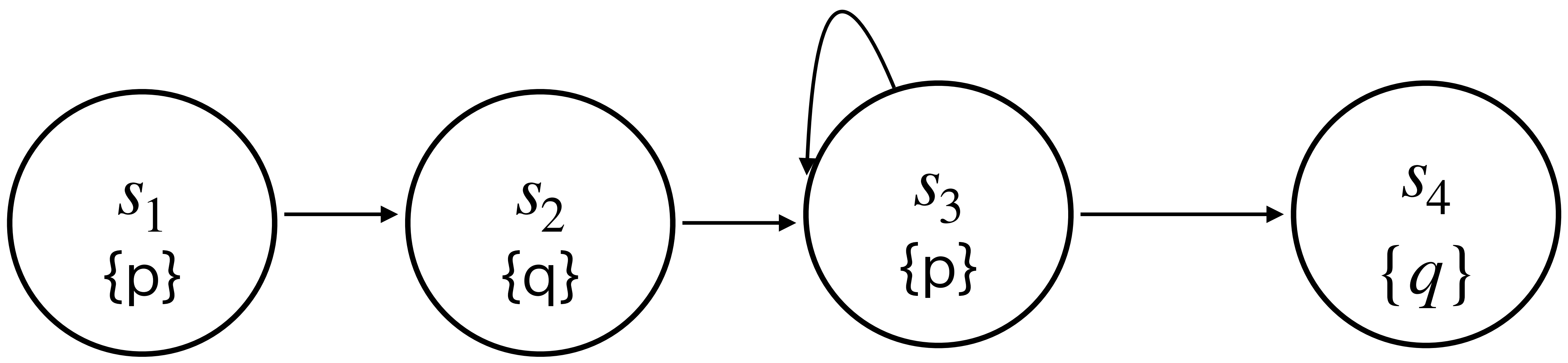

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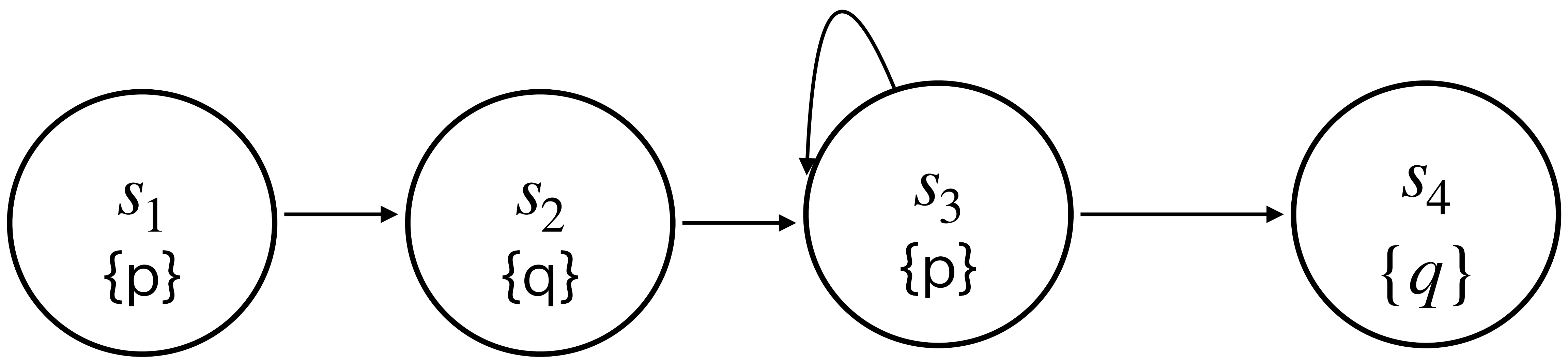

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$\pi_1 = p, q, p, p, p, \ldots$ $\pi_2 = p, q, p, p, p, p$ $< M > \models F$

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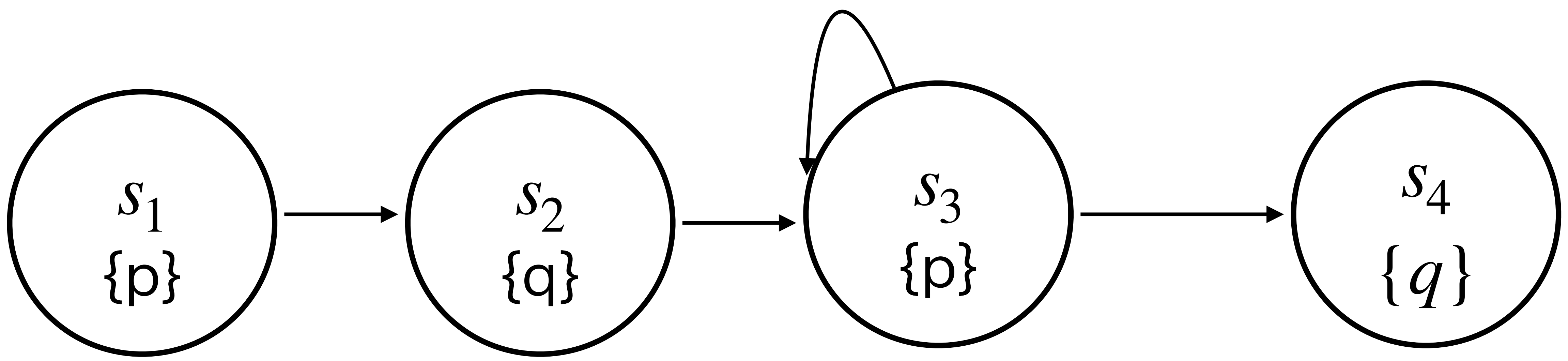

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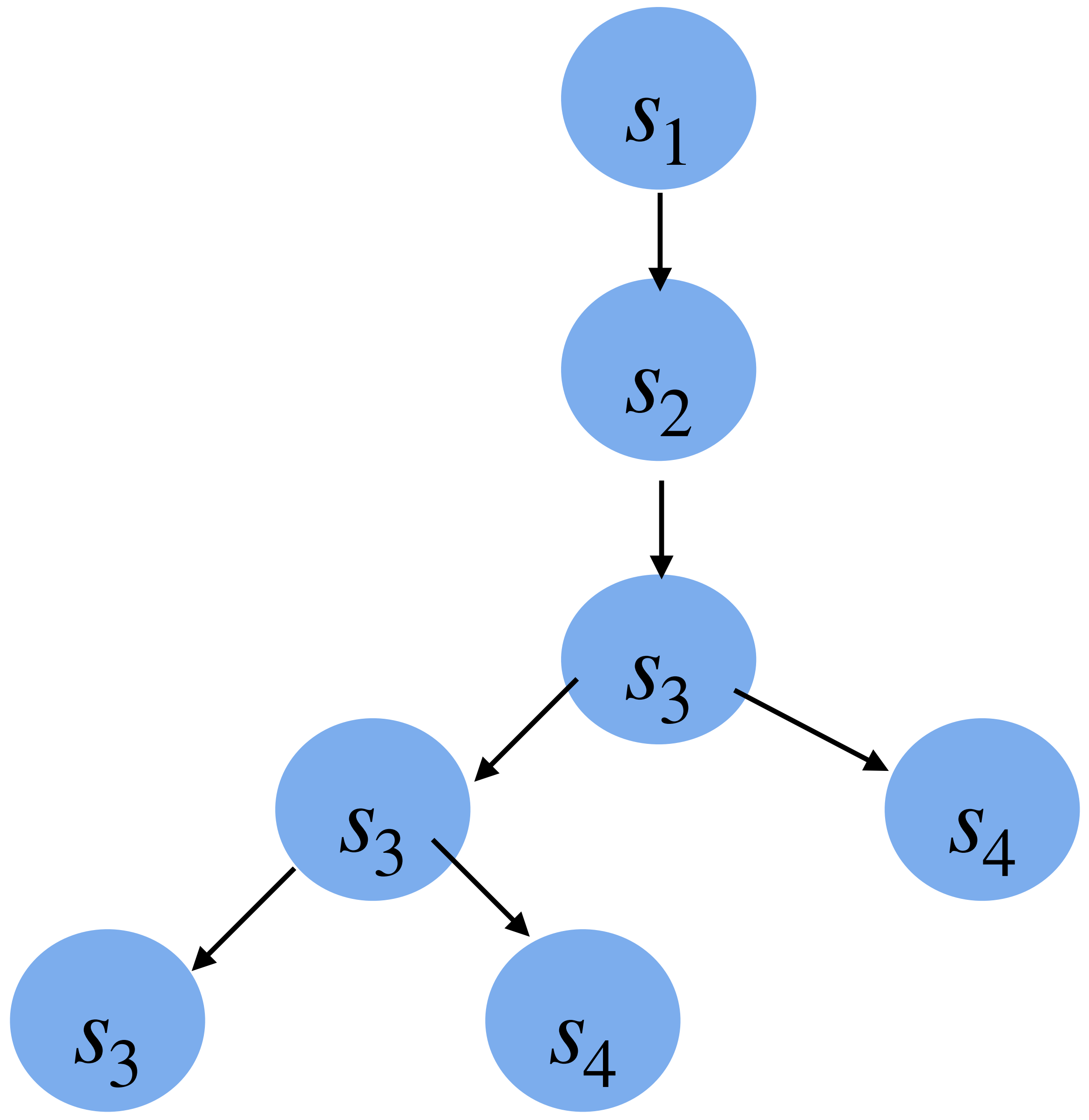

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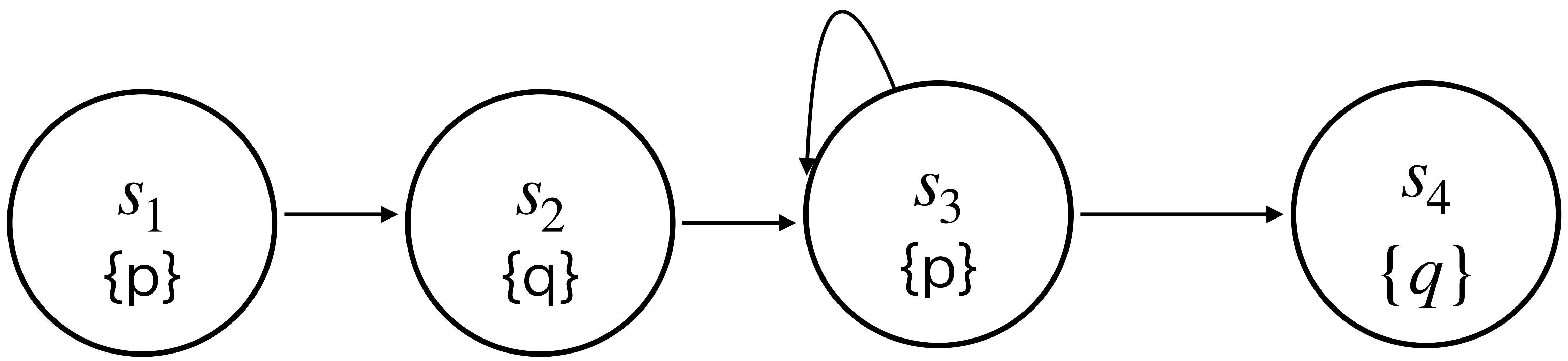

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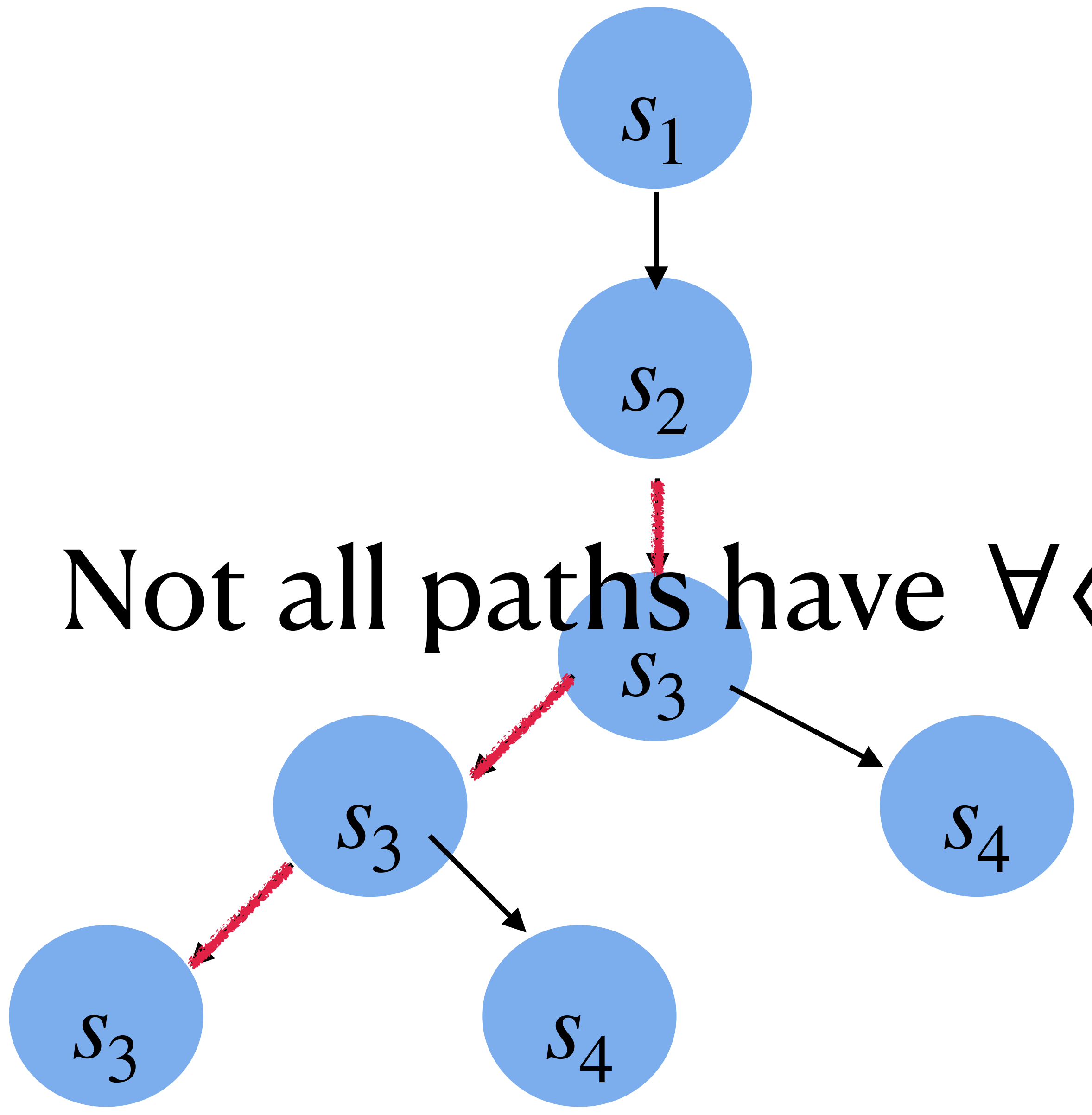
$$F_1 = \forall \Box (p \rightarrow \forall \Diamond q)$$

$$< M > \nvDash F_1$$



Not all paths have $\forall \Diamond q$

CTL :Examples

$$F_{LTL} = \Diamond \mathbf{N} a \qquad F_{CTL} = \forall \Diamond \forall \mathbf{N} a$$

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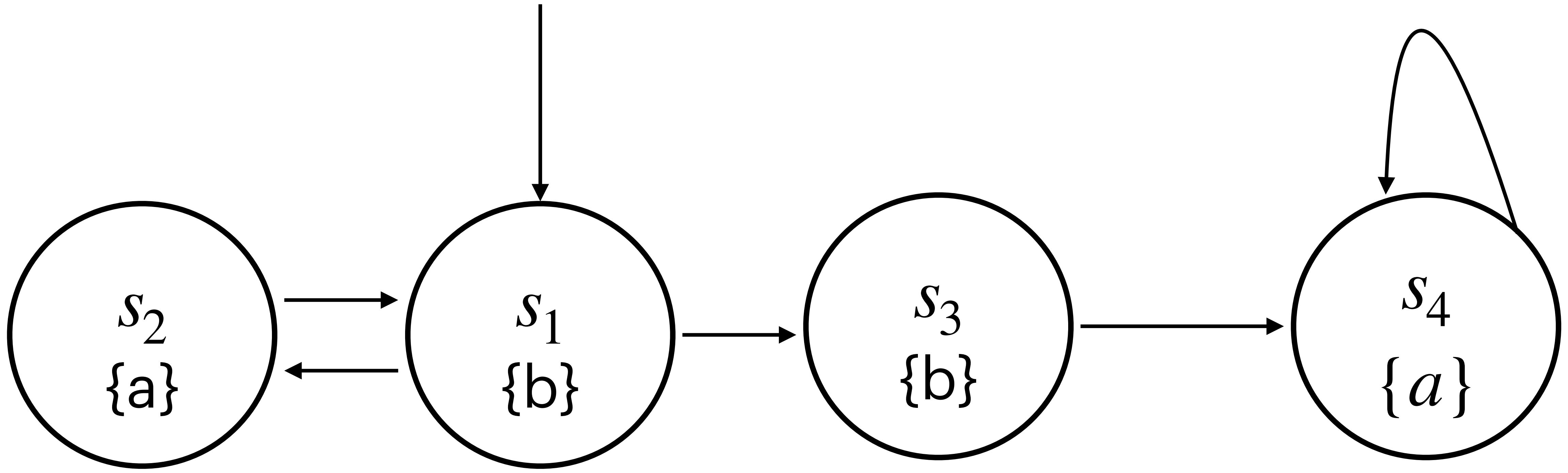

CTL :Examples

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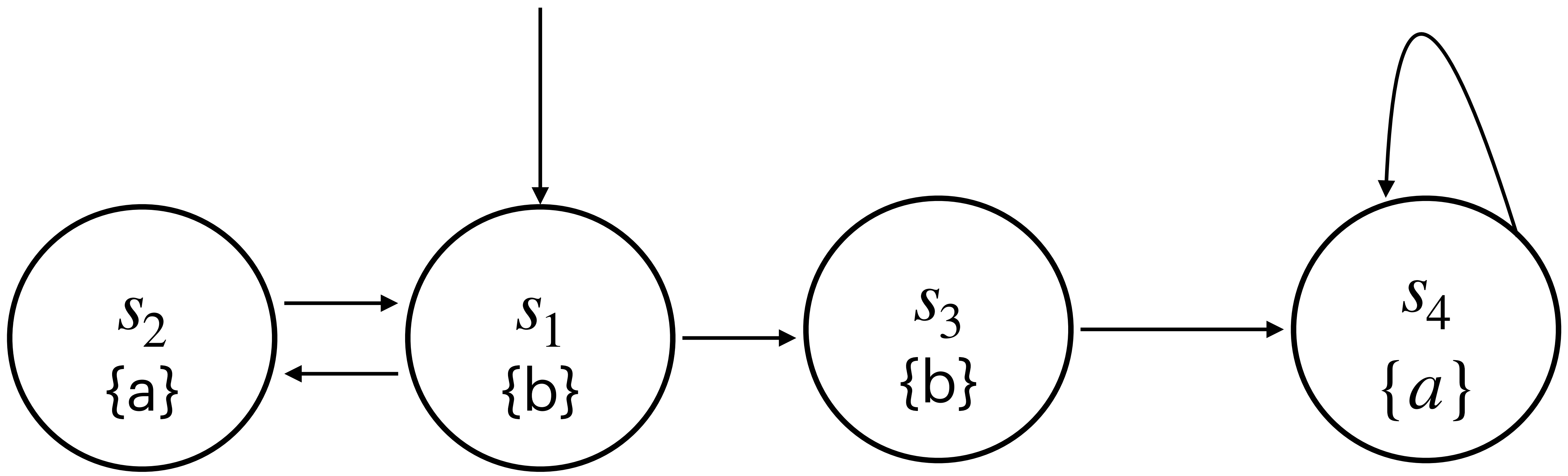
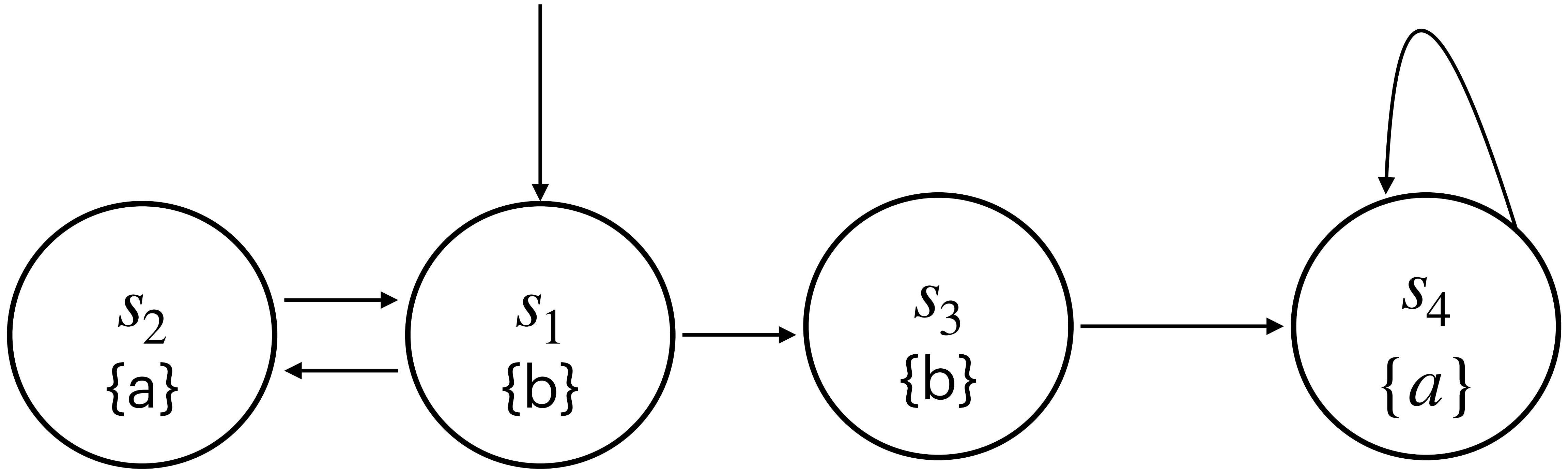


$M \vDash F_{LTL}$

$M \nvDash F_{CTL}$

CTL :Examples

$F_{LTL} = \Diamond \mathbf{N} a$ $\qquad$ $F_{CTL} = \forall \Diamond \forall \mathbf{N} a$



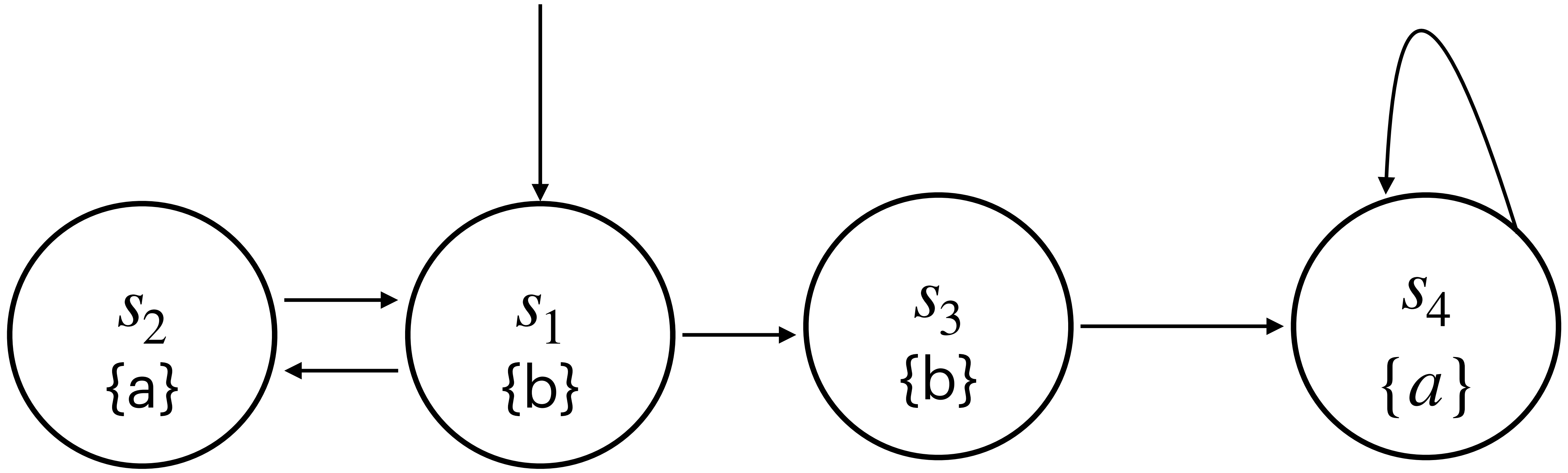
$M \models F_{LTL}$

$M \not\models F_{CTL}$

$\Diamond \mathbf{N} a \equiv \mathbf{N} \Diamond a$

CTL :Examples

$F_{LTL} = \Diamond \mathbf{N} a$ $\quad$ $F_{CTL} = \forall \Diamond \forall \mathbf{N} a$



$M \models F_{LTL}$

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$\Diamond \mathbf{N} a \equiv \mathbf{N} \Diamond a$ $\quad$ $\forall \Diamond \forall \mathbf{N} a \not\equiv \forall \mathbf{N} \forall \Diamond a$

CTL :Examples

$$\forall\Box\forall\Diamond a \stackrel{?}{\equiv} \Box\Diamond a$$ Yes!

Infinitely often a.

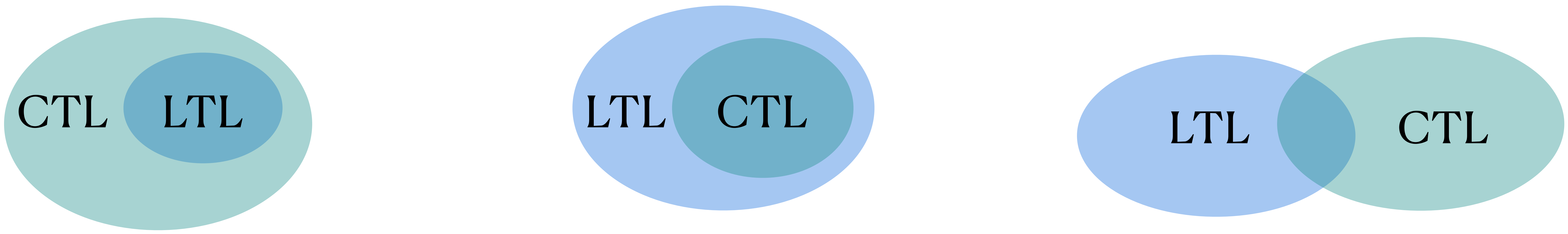
$$\forall(a\mathbf{W}b) \equiv a\mathbf{W}b$$

$$\forall\Diamond a \equiv \Diamond a$$

...

LTL vs CTL

LTL reasons about paths vs CTL reasons about states



LTL vs CTL

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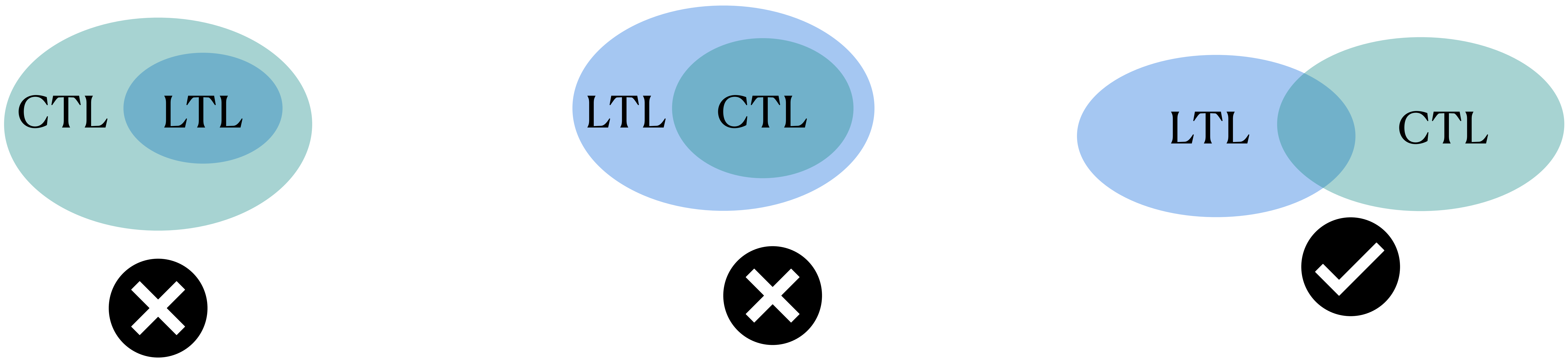
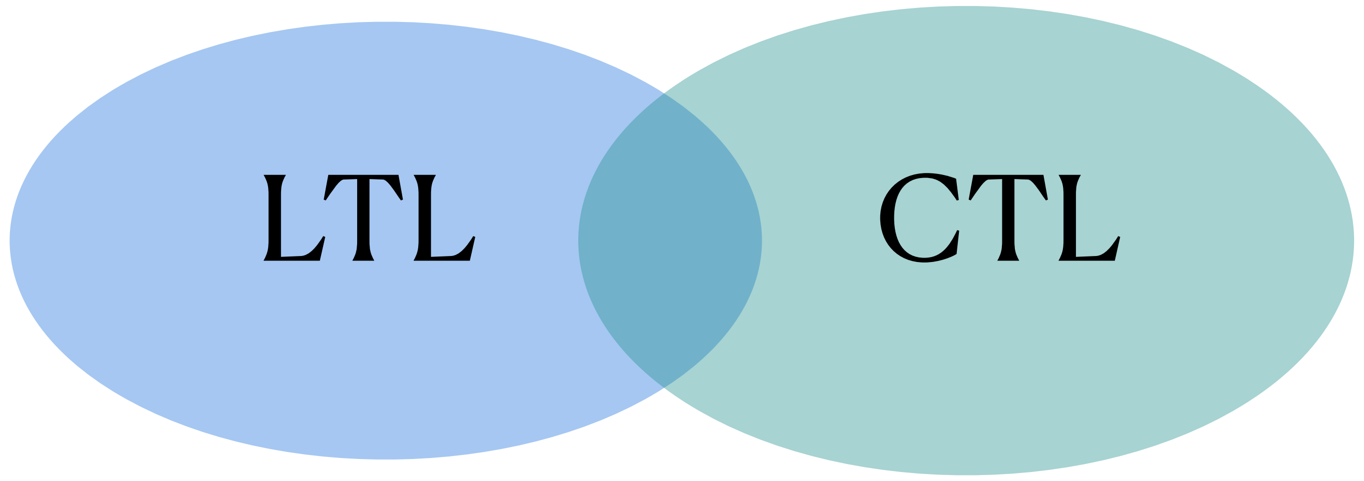


LTL vs CTL



LTL reasons about paths vs CTL reasons about states

Many CTL formula can't be expressed as LTL.

For those containing paths quantified existentially. $\forall \Box (p \rightarrow \exists \Diamond q)$

Many LTL formula can't be expressed as CTL.

Those that select a range of paths with a property.

$\Diamond p \rightarrow \Diamond q \qquad \Diamond \Box p$

$\Box \Diamond p \rightarrow \Box \Diamond q$

Course Webpage

Thanks!