COL:750

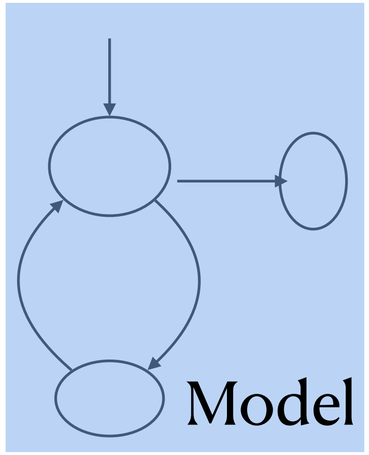
Foundations of Automatic Verification

Instructor: Priyanka Golia

Course Webpage 

https://priyanka-golia.github.io/teaching/COL-750/index.html

Does Code Satisfy Requirements ?

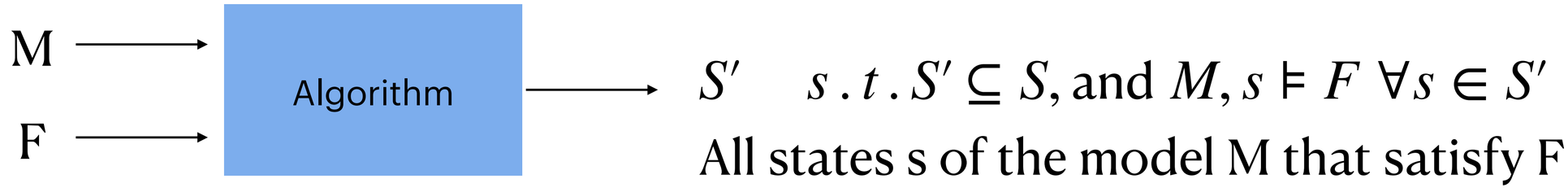


Does Model Satisfy Logical formulation: LTL/CTL Formula ?

Model Checking

Model Checking Algorithm

$$M, s \models F?$$



Note that not necessarily $I \subseteq S'$

Labelling Algorithm —

1. Does not scale well to large systems due to state explosion.
2. Memory-intensive as it maintains explicit labels for each state.

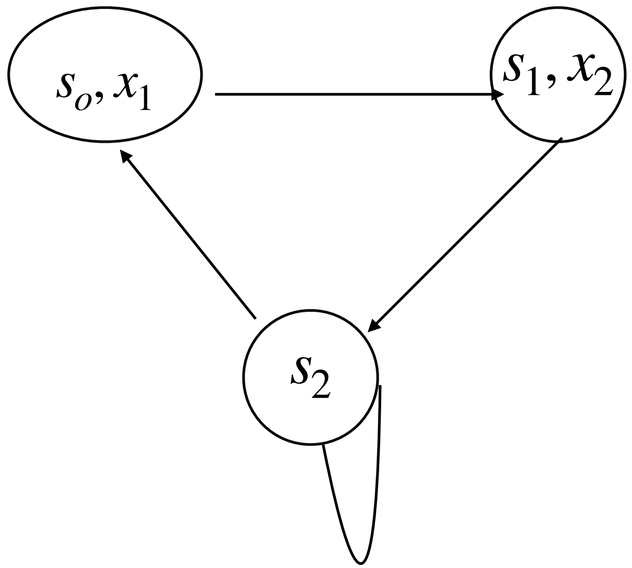
We need better data structure.

Implementing CTL Model Checking using BDDs

CTL model checking computes a set of states $[F_i]$ for every sub-formula F_i of the original formula F.

Sets of states will be represented using ROBDDs

That describes characteristic function of the set



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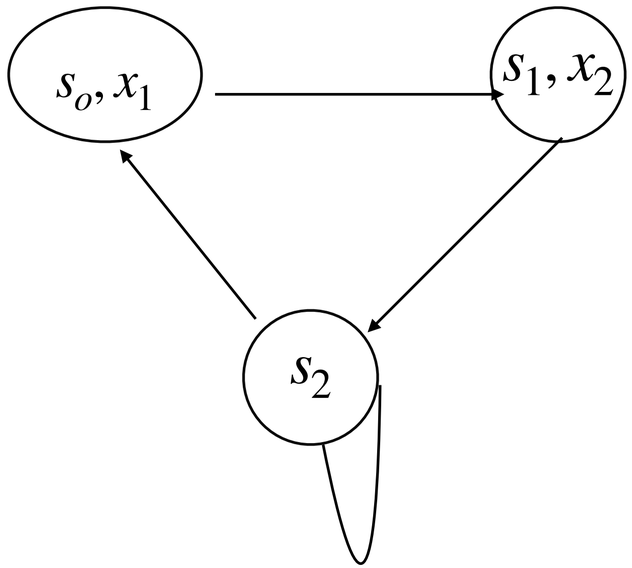

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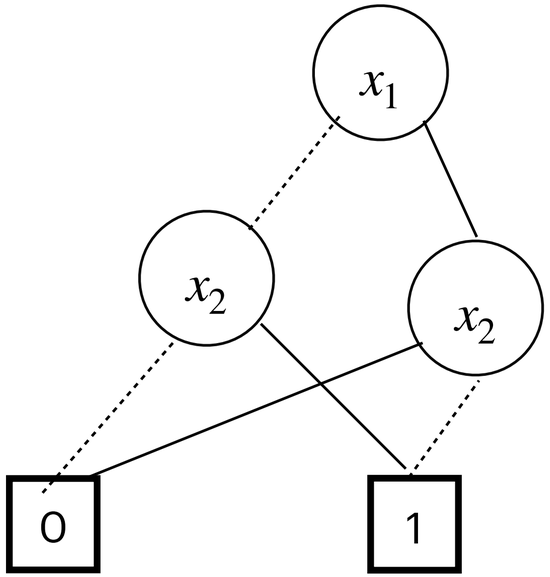


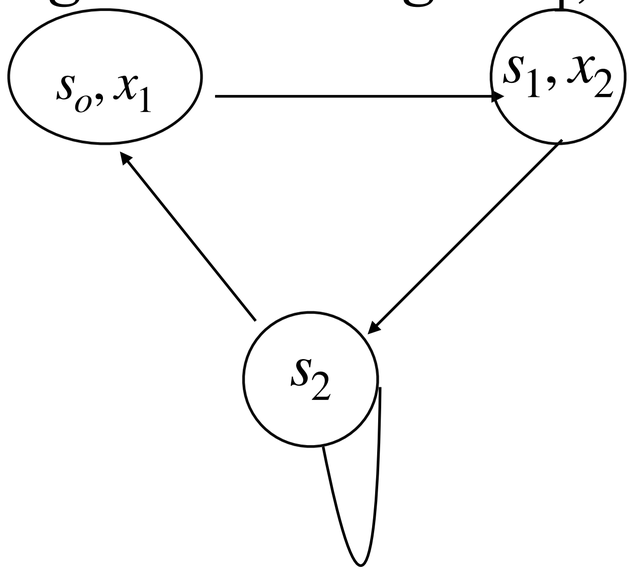
ROBDD for the set $\{s_o, s_1\}$

Set of states	Representation by	Representation by Boolean
∅		0
{so}	(1,0)	$x_1 \wedge \neg x_2$
{s1}	(0,1)	$\neg x_1 \wedge x_2$
{s2}	(0,0)	$\neg x_1 \wedge \neg x_2$
{s0,s1}	(1,0),(0,1)	$(x_1 \wedge \neg x_2) \vee (\neg x_1 \wedge x_2)$
{s0,s2}	(1,0),(0,0)	$(x_1 \wedge \neg x_2) \vee (\neg x_1 \wedge \neg x_2)$
{s1,s2}	(0,1),(00)	$(\neg x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2)$
{s0,s1,s2}	(1,0),(0,1),(0,0)	$(x_1 \wedge \neg x_2) \vee (\neg x_1 \wedge x_2) \vee (\neg x_1 \wedge \neg x_2)$

Implementing CTL Model Checking using BDDs

Representing the transition relations.

- Transition relations ($\rightarrow$) $\subseteq$ S $\times$ S are represented by ROBDDs on 2n variables.
- If the variables $x_1, \ldots, x_n$ describe the current state, and the variables $x'_1, x'_2, \ldots x'_n$ describe the next state.
- The good ordering is $x_1, x'_1, x_2, x'_2, \ldots, x_n, x'_n$ (interleaving).



X1	X2	X'1	X'2	->
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
1	0	0	1	1
0	0	0	1	0
..	..	..	..	..

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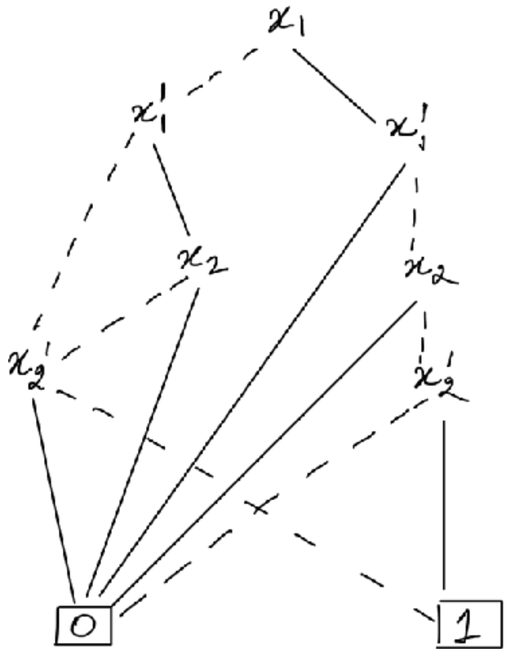
ROBDD of $F^{\rightarrow}$



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..	..	..	..	..

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But exploring Truth table will be expensive.

Can we learn $F^{\rightarrow}$ without Truth table?

X1	X2	X'1	X'2	->
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
1	0	0	1	1
0	0	0	1	0
..	..	..	..	..

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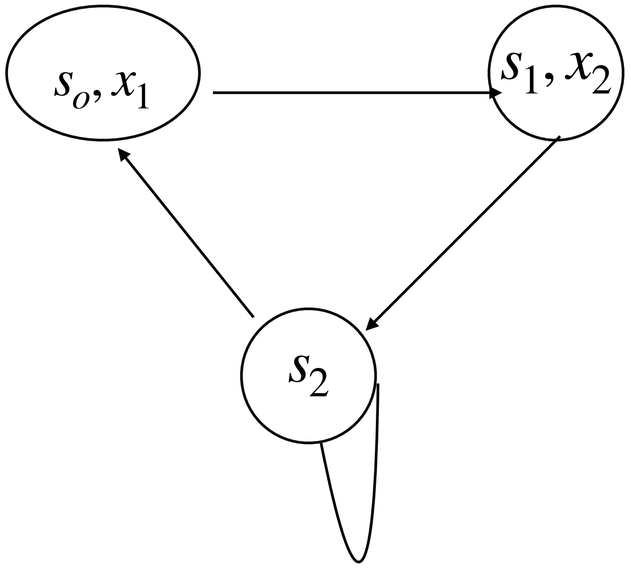


Can we learn $F^{\rightarrow}$ without Truth table?

$$F^{\rightarrow} := (x_1 \wedge \neg x_2 \wedge \neg x'_1 \wedge x'_2) \vee (\neg x_1 \wedge x_2 \wedge \neg x'_1 \wedge \neg x'_2) \vee (\neg x_1 \wedge \neg x_2 \wedge \neg x'_1 \wedge \neg x'_2) \vee (\neg x_1 \wedge \neg x_2 \wedge x'_1 \wedge \neg x'_2)$$

Convert $F^{\rightarrow}$ to ROBDD.

Implementing CTL Model Checking using BDDs

Symbolic Model Checking — it represents and manipulates sets of states and transitions using symbolic expressions or formulas (like Boolean functions or Binary Decision Diagrams) rather than explicitly enumerating each state.

Specification — $F = \exists \mathbf{N} p$

Pre([p]) same as Pre(Y)

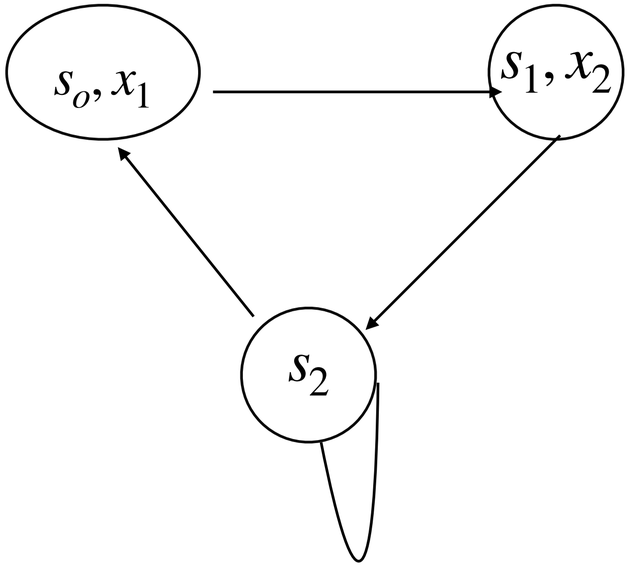
$B_{Pre(Y)} =$ exists $(X', \text{apply}(\wedge, F^{\rightarrow}, F_{Y'}))$

Where X' is set of next state variables.

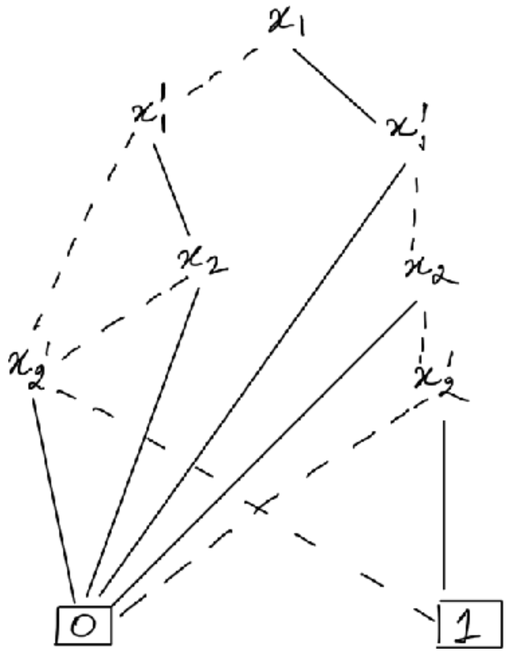
$F^{\rightarrow}$ is the ROBDD representing the transition relation.

$F_{Y'}$ is the ROBDD representing the set Y with variables $x_1, x_2, \ldots, x_n$ renamed to $x'_1, x'_2, \ldots, x'_n$

Symbolic Model Checking



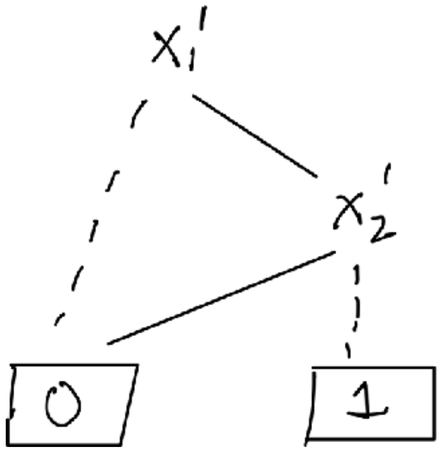
ROBDD of $F^{\rightarrow}$



$\exists \mathbf{N} x_1$

$B_{Pre(Y)} =$ exists $(X', \text{apply}(\wedge, F^{\rightarrow}, F_{Y'}))$

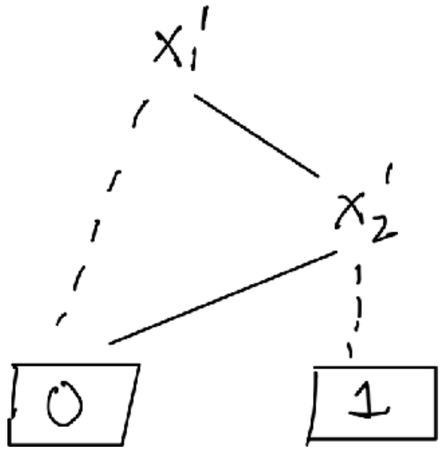
$F_{Y'} = ROBDD(s_0)$



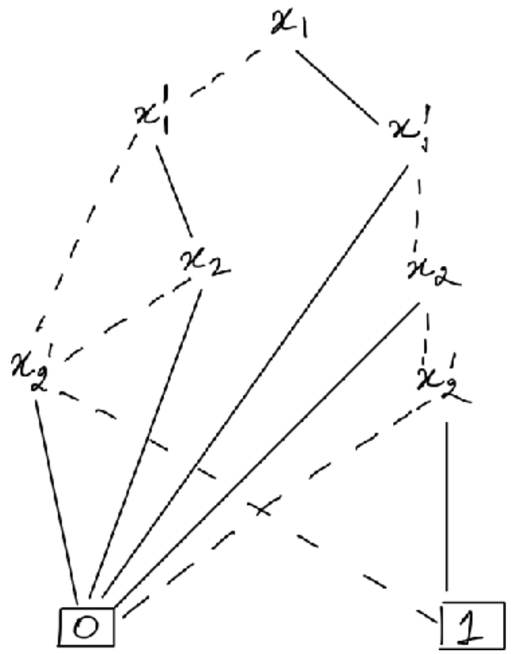
Symbolic Model Checking

$B_{Pre(Y)} =$ exists $(X', \text{apply}(\wedge, F^{\rightarrow}, F_{Y'}))$

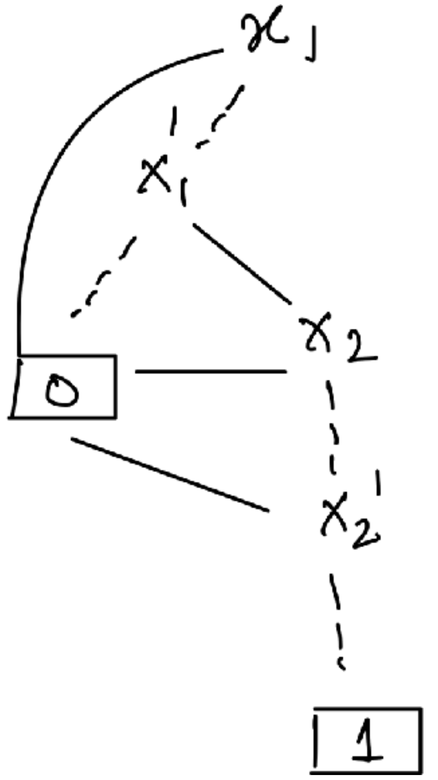
$B_{Pre(Y)} = exists(x'_1, x'_2, apply(\wedge, F^{\rightarrow}, F_{Y'})$



$F_{Y'} = ROBDD(s_0)$



ROBDD of $F^{\rightarrow}$



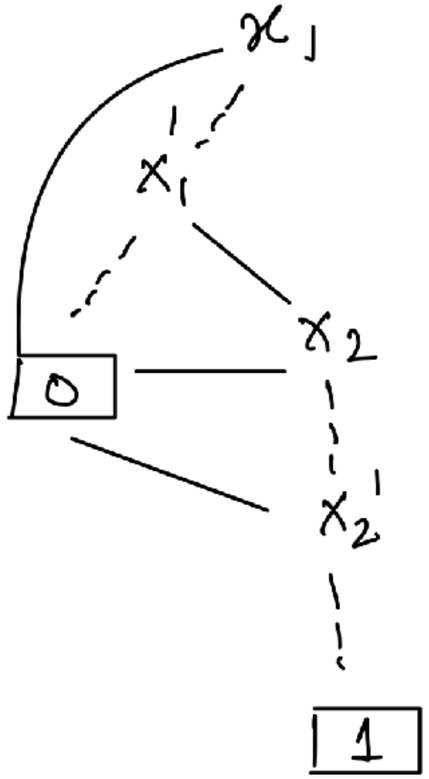
$apply(\wedge, F^{\rightarrow}, F_{Y'})$

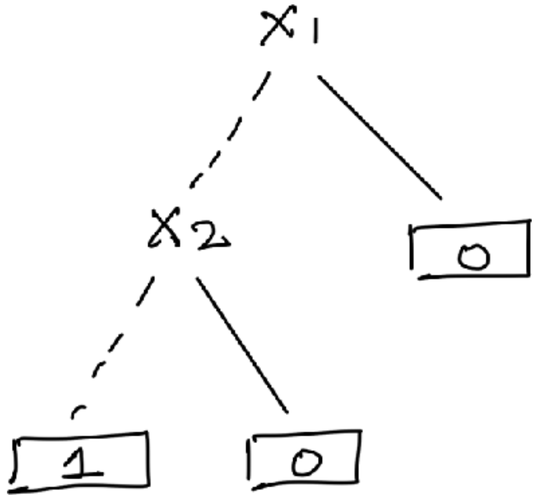
Symbolic Model Checking

$$B_{Pre(Y)} = exists(x_1', x_2', apply(\wedge, F^{\rightarrow}, F_{Y'})$$

$B_{Pre(Y)} =$

$restrict(x_1, x_2, ,0,0) \vee restrict(x_1, x_2,1,0) \vee restrict(x_1, x_2,0,1) \vee restrict(x_1, x_2,1,1)$





$restrict(x_1, x_2, F_1,1,0)$

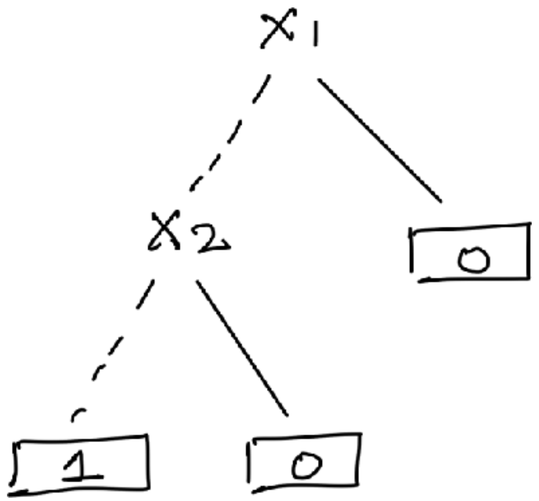
$restrict(x_1, x_2, F_1,0,0) \vee restrict(x_1, x_2, F_1,0,1)$

$restrict(x_1, x_2, F_1,1,1) = 0$

$$F_1 = apply(\wedge, F^{\rightarrow}, F_{Y'})$$

Symbolic Model Checking

$$B_{Pre(Y)} = exists(x'_1, x'_2, apply(\wedge, F^{\rightarrow}, F_{Y'}))$$



ROBDD of s_2

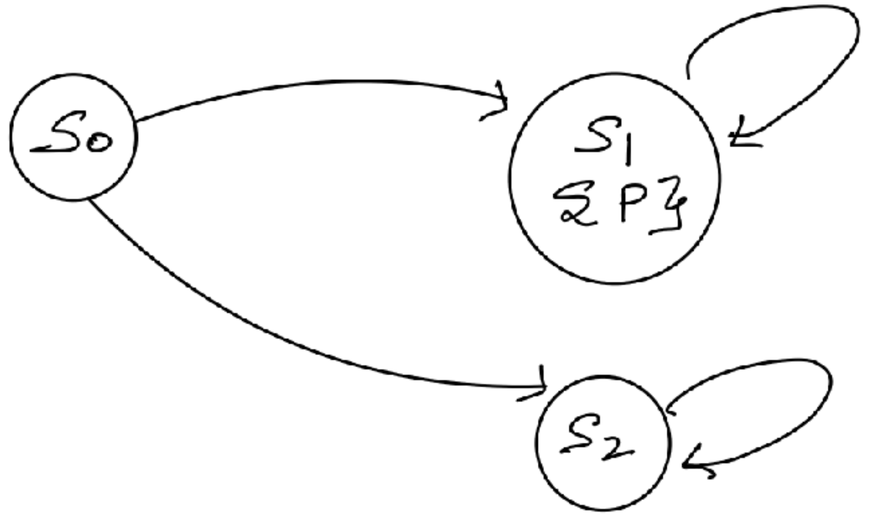
CTL Model Checking Algorithm —Symbolic Model Checking

Function Label(F, M){

Case F *of* :

True	return S
False	return {}
p	return $\{s \in S \mid p \in L(s)\}$
$\neg F_1$	return $\neg ROBDD$ *of* F_1
$F_1 \wedge F_2$	return $apply(\wedge, ROBDD(F_1), ROBDD(F_2))$
$\exists \mathbf{N} F_1$	return $\mathrm{pre}(ROBDD(F_1'), ROBDD(F^{\rightarrow}))$
$\exists \Box F_1$	return $Label\_EG(ROBDD(F_1'), ROBDD(F^{\rightarrow}))$
$\exists F_1 \mathbf{U} F_2$	return $Label\_EU(ROBDD(F_1'), ROBDD(F_2'), ROBDD(F^{\rightarrow}))$

End Case



$\exists \Diamond P$

$\cong \quad \exists [\text{true} \cup p]$

$x_1 x_2 \rightarrow$ for states

$S_0 = \quad \neg x_1 \wedge \neg x_2$

$S_1 = \quad \neg x_1 \wedge x_2$

$S_2 = \quad x_1 \wedge \neg x_2$

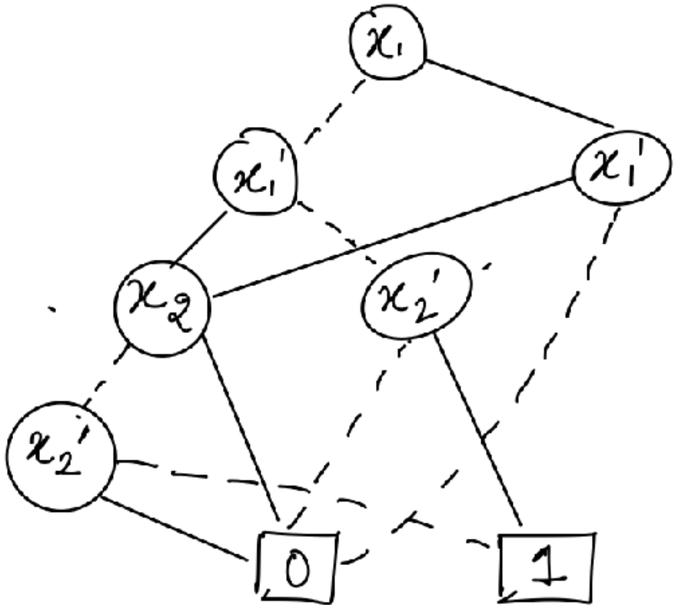
$P(x_1, x_2) = \neg x_1 \wedge x_2$

$$T: \quad (\neg x_1 \wedge \neg x_2 \wedge \neg x_1' \wedge x_2') \vee (\neg x_1 \wedge x_2 \wedge \neg x_1' \wedge x_2') \vee (\neg x_1 \wedge \neg x_2 \wedge x_1' \wedge \neg x_2') \vee (x_1 \wedge \neg x_2 \wedge x_1' \wedge \neg x_2')$$

$(S_0 \to S_1)$ $(S_1 \to S_1)$ $(S_0 \to S_2)$ $(S_2 \to S_2)$

$O = x_1, x_1', x_2, x_2'$

order



$\exists F_1 \cup F_2$

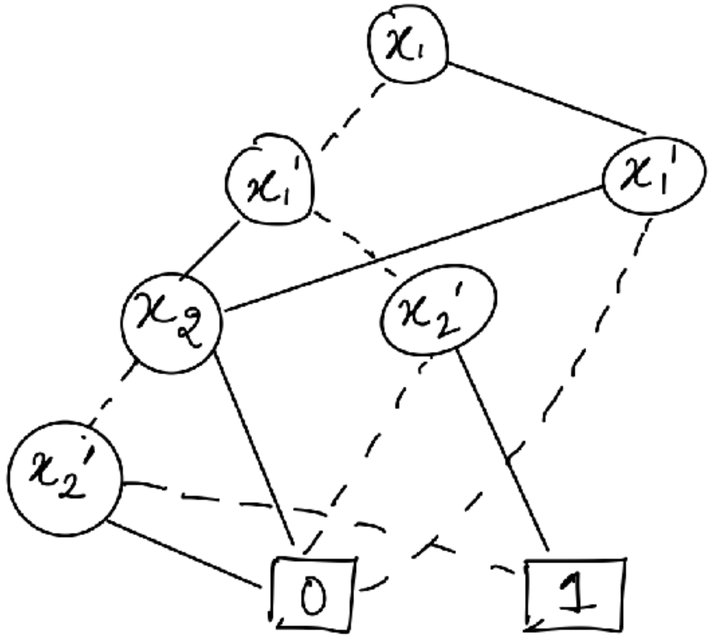
```
Label_EU([F1], [F2])
{
    X = [F2]
    Y = S
    while X ≠ Y do:
        Y = X
        X = X ∪ ([F1] ∩ pre(X))
    Return X
}
```

$\exists [true \cup p]$

pre([p])

$$\rightarrow \exists x_1' x_2' \cdot \left(T(x_1, x_2, x_1', x_2') \wedge P(x_1', x_2') \right)$$

$$\rightarrow \exists x_1' x_2' \cdot$$



$\wedge$