Assignment 2: Dynamic Epistemic Logic

Deadline: 14th April 2026, 11:59 pm

Part	Raw Marks	Course Contribution
Part 1 — Theory	40 / 80	Scaled to **20 marks**
Part 2 — Programming	40 / 80	
Part 3 — Bonus	10 (absolute)	Added directly to course total

Instructions

1. For Part-1 and Part-2, prepare a PDF report named `report.pdf` containing your solutions with proper explanation. You will be graded on the clarity of your explanations.
2. AI tools are strictly prohibited. Any use of AI assistance will result in a penalty as deemed fit by the course instructor.
3. Submit `report.pdf` and all other mentioned deliverables on Gradescope. Do not submit any files already present in the starter code.
4. In this assignment, we will study **Dynamic Epistemic Logic (DEL)**. Before attempting any problems, go through the materials provided on the course portal.
5. This assignment is not autograded. There will be a code demo and viva to evaluate your understanding. Note that the grading of the report will depend on the quality of your explanations.
6. For doubts, use Piazza. Send an email to Aneeket(cs1221116@iitd.ac.in) for doubts related to bonus section that may disclose implementation details.
7. The theoretical material and starter code can be found here.

Chuangtse and Hueitse had strolled onto the bridge over the Hao, when the former observed, "See how the small fish are darting about! That is the happiness of the fish." "You are not a fish yourself," said Hueitse. "How can you know the happiness of the fish?" "And you not being I," retorted Chuangtse, "how can you know that I do not know?"

— Chuangtse, c. 300 B.C.

Part 1 — Theory [40 marks]

Go through the provided documents to understand the fundamentals of Epistemic Logic and then answer the following questions.

Problem 1.1 Three Vindy Men [6 marks]

There are three Vindy men - Arunabh, Dinu and Sabhya. It is common knowledge that there are 3 red hats and 2 white hats. Aneeket places a hat on each Vindy man's head and asks them sequentially if they know the color of their own hat. Arunabh says he does not know; Dinu also says he does not know; Sabhya says he knows.

(a) [2] What color is Sabhya's hat? Justify your answer.

(b) [2] Construct the Kripke structure representing the initial state of the problem. Demonstrate how the structure changes after Arunabh announces that he is unable to determine the answer, and then again after Dinu declares that he cannot determine the answer as well.

(c) [2] Now if Sabhya is wearing a blindfold, construct the Kripke structure representing the initial state of the problem.

Problem 1.2 Muddy Children [17 marks]

Suppose n children are playing in the park. Pradyuman has told them that if they get dirty there will be severe consequences. So, of course, each child would prefer to keep clean, but each would love to see the others become dirty. During play, it happens that k of the children get mud on their foreheads. Each child can see the others' foreheads but not their own. At this point, Tandon enters the scene and announces: "At least one of you has mud on your forehead." *He then asks repeatedly:* "Does any of you know whether you have mud on your own forehead?" *Assume that all children are perceptive, perfectly rational, honest, and answer together.*

(a) [2] What will finally happen? Describe the initial Kripke structure and explain how it changes after each of Tandon's announcements.

(b) [3] Now consider the case when all children have bandages on their eyes and thus can't see and that this is common knowledge. Draw the Kripke structure before and after Tandon speaks.

(c) [9] Now suppose it is the case that all children, *except perhaps child 1* stay attentive when Tandon is speaking. It is common knowledge that Tandon says either "At least one of you has mud on your forehead" or a vacuous statement such as "Two plus two is four". Therefore, even when inattentive, child 1 who did not hear Tandon speaking knows that Tandon made one of the two statements.

   i. [3] Explain the state of the Kripke structure after Tandon's statement. For the case when $n = 2$, draw the Kripke structure.

ii. [3] Is it possible for the children to ascertain whether they are muddy?

iii. [3] Can the children determine whether they are muddy if instead Tandon initially says *"Two or more of you have mud on your forehead"*?

(d) [3] Suppose in the beginning, instead of *"At least one of you has mud on your forehead."*, Tandon says *"Child 1 has mud on his forehead"*. Explain why the other children *cannot* deduce whether they are muddy, even though they apparently have more information.

Problem 1.3 Aces and Eights [17 marks]

The Aces and Eights game is played with a deck of four aces and four eights, with three players. Six cards are dealt (two each); the remaining two are left face-down. Each player holds their cards face-out (others can see them, the holder cannot). Players take turns trying to determine their own cards; if a player does not know, they must say so. You are playing with Aneeket and Raj.

(a) [1] How many possible worlds are there if the *suit* of the card matters?

(b) [1] How many possible worlds are there if suits are *ignored*?

(c) [2] If we choose to ignore the suits, construct the Kripke structure representing the initial state.

(d) [2+2=4] Aneeket (first) holds two aces; Raj (second) holds two eights. Both say they cannot determine their cards. What cards do you hold? When considering the Kripke structure, which edges disappear from the when you learn that Aneeket and Raj cannot determine their cards?

(e) [2+2=4] You go first. Aneeket (second) holds two eights; Raj (third) holds one ace and one eight. No one determines their cards on their first turn. What do you hold? Also show which edges disappear from the initial Kripke structure in this case.

(f) [2] You go second. Aneeket (first) holds one ace and one eight; Raj (third) also holds one ace and one eight. No one determines their cards on the first turn; Aneeket cannot determine his on his second turn either. What do you hold?

(g) [3] Show that someone will *always* be able to determine their cards. Then exhibit a situation where exactly one player can determine their cards, and the other two can never do so regardless of how many rounds are played.

Part 2 — Programming [40 marks]

Programming Environment Setup

- All programs must be written in Python.
- **For each programming problem, also provide a brief explanation of your modeling decisions in `report.pdf`.**
- Install the following packages for visualisation:
```
sudo apt install graphviz dot2tex libtinfo6 texlive-latex-base \
  texlive-latex-extra texlive-fonts-recommended poppler-utils \
  preview-latex-style texlive-pstricks
```
- We will work with two model checkers: `demo_s5` and `demo_light`.
- Study `demo_s5/cheryl.py` and `demo_light/muddy.py` as examples of how to model problems in the `demo_s5` and `demo_light` model checkers respectively. Run them as:
```
python3 cheryl.py
python3 muddy.py
```
`cheryl.py` imports methods from `kripkevis.py` which creates `/tmp` directory in the current directory and in that, using its `pdf_model` method, generates the `.pdf` files showing the state of the Kripke structure.

Problem 2.1 Understanding the Tools [19 marks]

Study how `demo_s5` and `demo_light` work by reading the corresponding source files.

(a) [8=4+4] Explain how `demo_s5` and `demo_light` model and solve a DEL problem.

(b) [4] Based on your analysis, explain what makes `demo_light` **more powerful** than `demo_s5` in terms of representational power.

(c) [7=3+4] **Theory of Mind & the Sally–Anne Test.**

Theory of Mind is the cognitive ability to recognize that other people have their own thoughts, beliefs, desires, and intentions, which may differ from both one's own perspective and from reality. A Theory of Mind test is a psychological assessment to determine whether this ability has developed. A key indicator of Theory of Mind is understanding that another person can hold a false belief. The Sally–Anne Test specifically evaluates whether a subject can distinguish between their own knowledge of reality and what another person mistakenly believes to be true.

In the scenario, the subject observes two characters, Sally and Anne. Sally places a toy in her basket and then leaves the room. While she is away, Anne moves the toy from the basket into her box. The subject is then asked: "When Sally returns, where will she look for her toy?" If the subject answers "the basket," they pass the test, demonstrating an understanding that Sally still holds an outdated and therefore false belief about the toy's location. If the subject answers "the box," they fail, indicating that they are projecting their own knowledge of the situation onto Sally and cannot yet separate their perspective from hers. Children under the age of four generally fail this test.

i. [3] Can the Sally–Anne test be modeled using `demo_s5`? Justify your answer.

ii. [4] Can it be modeled using `demo_light`? If yes, write a program `sally_anne.py` that models the scenario and can be verified with `demo_light`.

Deliverables: `sally_anne.py`

Problem 2.2 Sum and Product [5 marks]

Anuj announces to logicians Vraj and Mhaske: "I have chosen two integers x, y such that $1 < x < y$ and $x + y \leq 100$. I will inform Vraj only of $s = x + y$, and Mhaske only of $p = xy$. These announcements remain private. The two of you are required to determine your numbers x and y." Anuj then proceeds as he said. The following conversation ensues:

i. Mhaske says: "I do not know it."

ii. Vraj says: "I knew you didn't."

iii. Mhaske says: "I now know it."

iv. Vraj says: "I now also know it."

(a) [4] Write a `demo_s5`-checkable program `sum_and_product.py` to solve and determine the pair (x, y).

(b) [1] Does the first announcement by Mhaske provide any useful information? Justify.

Deliverables: `sum_and_product.py`

Problem 2.3 Dining Cryptographers [10 marks]

Three cryptographers - Sanyam, Satwik and Abhijeet, learn that their dinner bill will be paid anonymously - either by one of them or by the NSA. They carry out the following protocol to determine who *is paying without compromising anonymity: each cryptographer flips a coin (shared only with their right neighbor). Each then states aloud whether the two coins they can see fell on the* same *or* different *sides; a paying cryptographer states the* opposite. *An odd number of "different" reports indicates a cryptographer paid; an even number indicates the NSA paid.*

(a) [2] Argue why the protocol is correct.

(b) [4] Write a program `dining_cryptos.py` modeling the above in `demo_s5`. The program must generate `m0.pdf` (initial Kripke structure) and `m1.pdf` (final Kripke structure).

(c) [4=2+2] By analyzing `m1.pdf`, verify both properties:

  i. [2] The protocol correctly reveals whether NSA or a cryptographer paid.

  ii. [2] If a cryptographer paid, the other two cannot determine which one.

Deliverables: `dining_cryptos.py`, `m0.pdf`, `m1.pdf`

Problem 2.4 Unexpected Hanging [6 marks]

A prisoner is sentenced by a judge: "You will be executed next week, but the day will be a surprise to you." The prisoner reasons - "The execution cannot be on Friday, because then it would not be a surprise. However, if it isn't on Friday, then it cannot be on Thursday either, as that would then no longer be a surprise, And so on. Thus, I will not be executed at all." Unfortunately, he is surprised by a Wednesday execution.

(a) [4] Write a program `unexpected_hanging.py` checkable by `demo_light` that creates a 5-state epistemic model (Mon–Fri) for the prisoner. Requirements:

- The prisoner has total initial uncertainty (all states connected).
- Hardcode the execution day as Wednesday.
- Define the safe passage of each day as a public announcement.
- Apply announcements chronologically from Monday night to Thursday night using `upd`.
- Include in `report.pdf`: images showing the evolution of the Kripke structure, and an explanation of what happens *after Wednesday passes* and what this means.

(b) [2] Suppose that professor Priyanka, had only said to the class that the quiz would take place next week, but without saying that the quiz would come as a surprise. Later, Abhinav walks past her office and overhears the teacher saying to TA Raj, "I am going to give the class a quiz next week, and the day of the quiz will be a surprise to them." The professor did not realize that Abhinav was overhearing her. What can Abhinav conclude on the basis of this information about the day of the quiz? But the plot thickens. Because after lunch break Abhinav says to the professor, "I heard you say that the day of our quiz next week will come as a surprise." The professor confirms this. However, later during the day, the teacher meets TA Raj again and tells him that Abhinav had overheard them earlier that day, and says, "But the day of the quiz will still come as a surprise!" Unfortunately, Abhinav overhears this again. What can Abhinav now conclude about the day of the quiz?

Deliverables: `unexpected_hanging.py`

Part 3 — Bonus [10 marks absolute]

Important Notice. Solving each problems contained in this part will earn you marks beyond the course total. This also implies that these are significantly harder than the problems presented above. If you claim to solve any part of these problems, you will be asked to deliver a presentation explaining your approach. If we find that your understanding of the solutions you are presenting is superfluous, you will receive a zero in the bonus and a penalty of 3 marks from the course total. Therefore, will we encourage and appreciate your engagement with these problems, it is for your own benefit to participate honestly.

Problem 3.1 Rumor grew of a shadow in the East, whispers of a nameless fear... [5 marks]

In the uneasy years when whispers first spread that the Shadow in the East was stirring again, the Free Peoples began to keep watch more carefully over their borders. Across the West there were n watch-posts—in Gondor's towers, the halls of the Dwarves, the woodland paths of the Elves, and the scattered villages of Men. Each watch-post had received one report concerning strange movements, dark rumors, or signs of Sauron's servants abroad. At the beginning, every watch-post knew only the report it had gathered itself. From time to time, two riders would meet on the road, sent from their respective posts to exchange tidings. When they met, they shared all the reports known to their watch-posts so far. After the meeting, both riders returned bearing the full collection of news that had been exchanged, so that their watch-posts now knew everything that had been shared between them. The leaders of the West wished to spread these warnings quickly, but perfect knowledge everywhere was not necessary at once. However, there is also a certain degree of mistrust between these people. So they would not want one to have more total knowledge of the growing threat than the others.

(a) [5] If the riders' meetings can be planned carefully, let the smallest number of such meetings needed before every watch-post knows exactly k reports be $F(n, k)$. Write a program `bonus_1.py` to compute $F(n, k)$ using any of the provided model checkers. You will be graded on the correctness and speed of your program.

Deliverables: `bonus_1.py`

Problem 3.2 Marked Foreheads [5 marks]

Samyak, Yash, and Narware each have a positive integer (x, y, and z respectively) written on their forehead. Each of them can see the others' numbers but not their own. It is common knowledge that exactly one number is the sum of the other two. They then make the following truthful announcements sequentially:

i. Samyak: "Really, I do not know my number."

ii. Yash: "Well I don't know mine either."

iii. Narware: "By Jove, I have no idea what mine is"

iv. Samyak: "Too bad for y'all, I know my number. It is x."

(a) [3] For a given value of x, can Yash and Narware figure out their numbers? (Hint: Being a student of computer science, you immediately realise that in the present form, the answer is not computable. In order to be able to write a program to solve this, the domain must be finite. You can therefore choose an upper bound ub such $x, y, z \leq ub$). Using any of the provided model checkers, write the program `bonus_2.py` to solve this.

(b) [2] For which upper bound is it the case that after the three ignorance announcements by Samyak, Yash, and Narware, Samyak always knows his number?

Deliverables: `bonus_2.py`

End of Assignment 2