# A Data Driven Approach for Boolean Functional Synthesis


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Joint work with: Friedrich Slivovsky [3], Subhajit Roy [2] and Kuldeep S. Meel [1]





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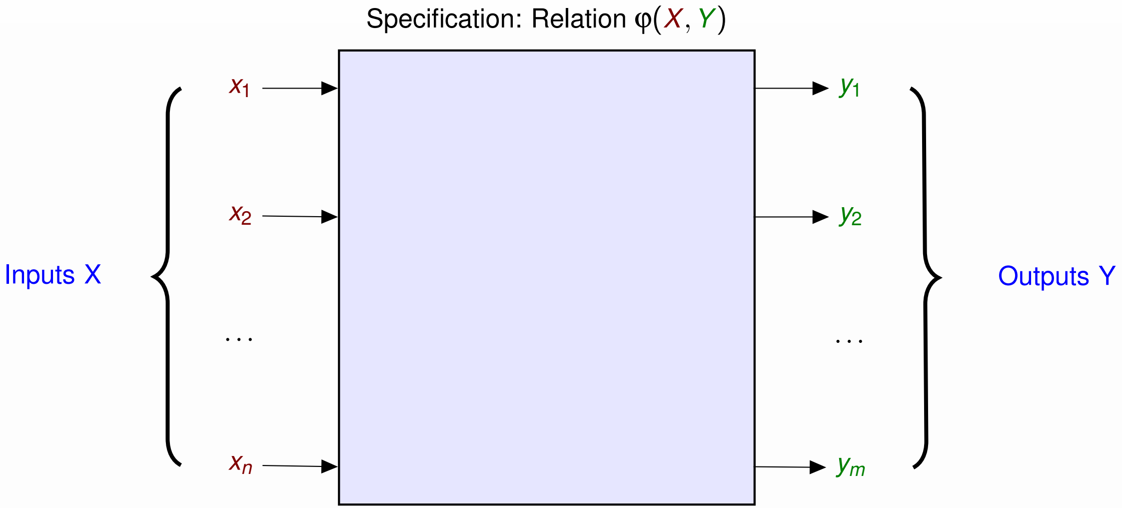
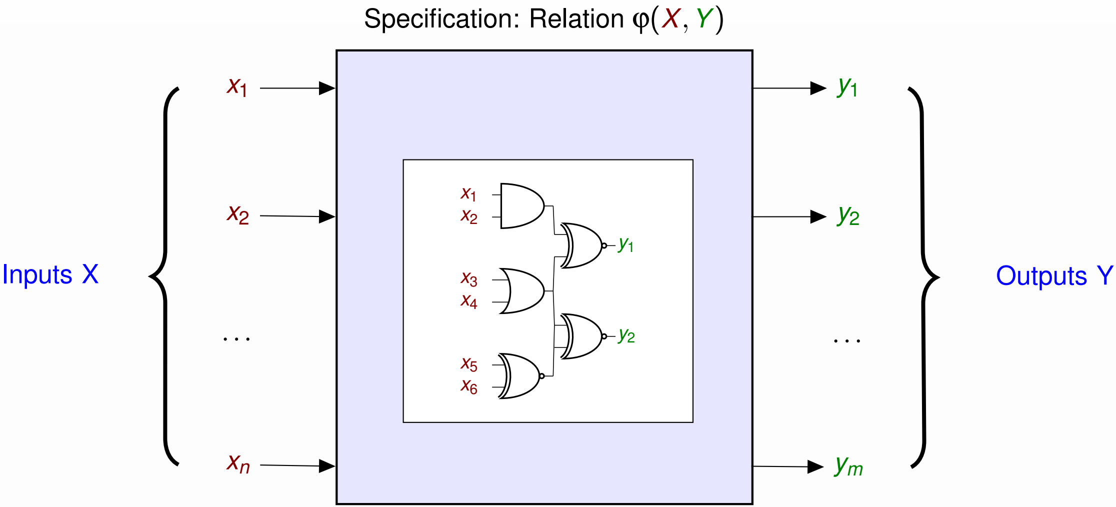
Corresponding Papers: CAV-20, ICCAD-21 (Best Paper Award Nomination)

# Synthesis

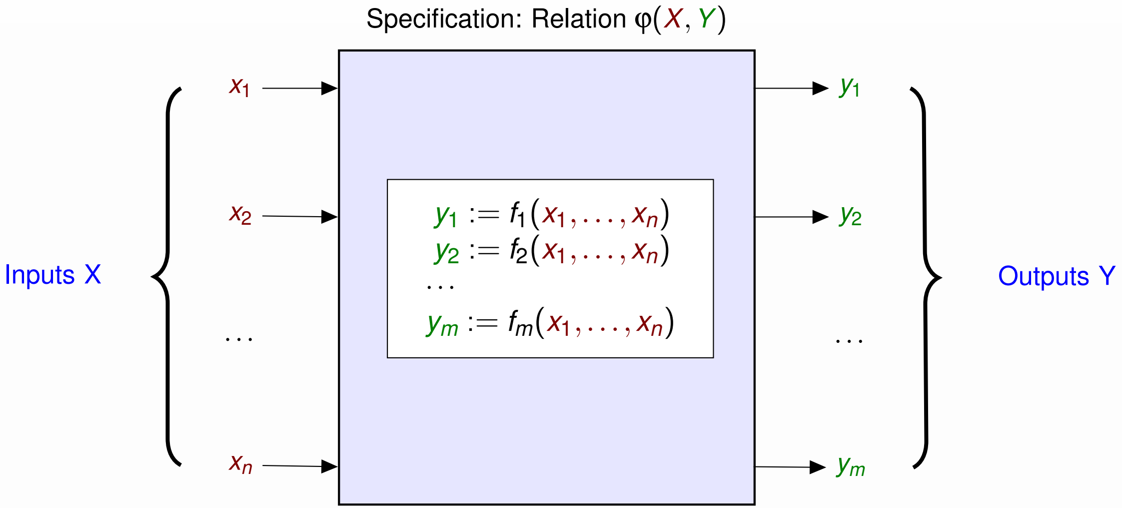


Specification: Relation $\varphi(X, Y)$
Inputs X
$x_1$
$x_2$
...
$x_n$
Outputs Y
$y_1$
$y_2$
...
$y_m$

# Synthesis

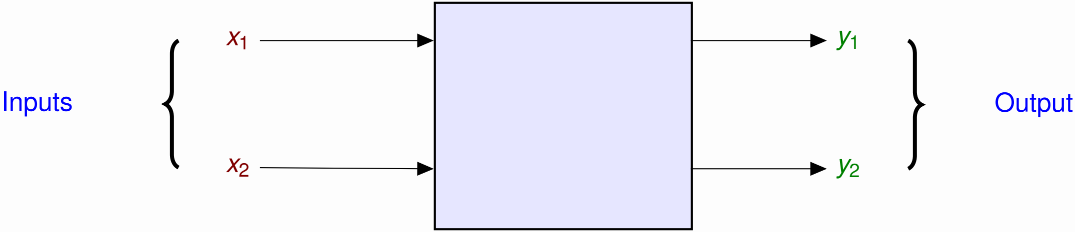


Specification: Relation φ(X, Y)
Inputs X
x1
x2
. . .
xn
x1
x2
x3
x4
x5
x6
y1
y2
y1
y2
. . .
ym
Outputs Y


Specification: Relation $\varphi(X, Y)$
$x_1$
$x_2$
$\dots$
$x_n$
Inputs X
$y_1 := f_1(x_1, \dots, x_n)$
$y_2 := f_2(x_1, \dots, x_n)$
$\dots$
$y_m := f_m(x_1, \dots, x_n)$
$y_1$
$y_2$
$\dots$
$y_m$
Outputs Y

# Synthesis – Example

$$\varphi(X, Y) = (y_1 \geq x_1) \wedge (y_1 \geq x_2) \wedge ((y_1 = x_1) \vee (y_1 = x_2))$$
$$\wedge (y_2 \leq x_1) \wedge (y_2 \leq x_2) \wedge ((y_2 = x_1) \vee (y_2 = x_2))$$



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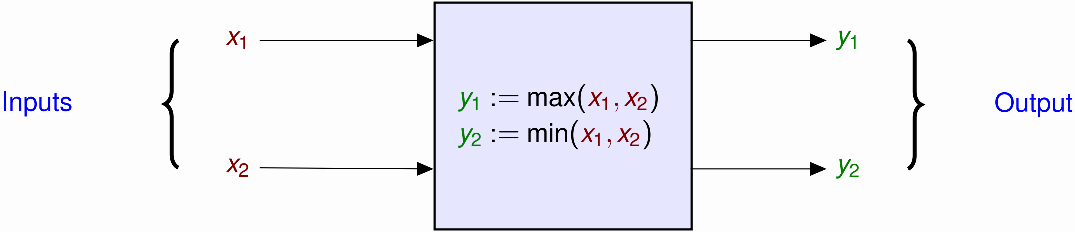

# Functional Synthesis

**Given** $\varphi(X, Y)$ over inputs $X = \{x_1, x_2, \ldots, x_n\}$ and outputs $Y = \{y_1, y_2, \ldots, y_m\}$.

**Synthesize** A function vector $F = \{f_1, f_2, \ldots, f_m\}$, such that $y_i := f_i(x_1, \ldots, x_n)$ such that:

$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

Each $f_i$ is called Skolem function and $F$ is called Skolem function vector.

Key Challenge: $\varphi(X, Y)$ is a relation

# Non-uniqueness of Skolem Functions

Let $X = \{x_1, x_2\}$, $Y = \{y_1\}$ and $\varphi(X, Y) = x_1 \vee x_2 \vee y_1$

Possible Skolem function: $f(x_1, x_2) := \neg(x_1 \vee x_2)$

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Possible Skolem function: $f(x_1, x_2) := \neg(x_1 \vee x_2)$

$$\varphi(X, F(X)) = x_1 \vee x_2 \vee (\neg(x_1 \vee x_2))$$

| $X$ | $\exists Y \varphi(X, Y)$ | | $\varphi(X, F(X))$ |
|---|---|---|---|
| $x_1 = 0, x_2 = 0$ | $y_1 = 1$ | True | True |
| $x_1 = 0, x_2 = 1$ | $y_1 = 1$ | True | True |
| $x_1 = 1, x_2 = 0$ | $y_1 = 1$ | True | True |
| $x_1 = 1, x_2 = 1$ | $y_1 = 1$ | True | True |

$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

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$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

Other possible Skolem functions: $f_1(x_1, x_2) = \neg x_1$ $\quad f_1(x_1, x_2) = \neg x_2$ $\quad f_1(x_1, x_2) = 1$

# Diverse Approaches

- From the proof of validity of $\forall X \exists Y \varphi(X, Y)$
  (Bendetti et al., 2005)
  (Jussilla et al., 2007)
  (Heule et al., 2014)
- Quantifier instantiation in SMT solvers
  (Barrett et al., 2015)
  (Bierre et al.,2017)
- Input-Output Separation
  (Chakraborty et al., 2018)
- Knowledge representation
  (Kukula et al., 2000 )
  (Trivedi et al., 2003)
  (Jiang, 2009)
  (Kuncak et al., 2010)
  (Balabanov and Jiang, 2011)
  (John et al., 2015)
  (Fried, Tabajara, Vardi, 2016,2017)
  (Akshay et al., 2017,2018)
  (Chakraborty et al., 2019)
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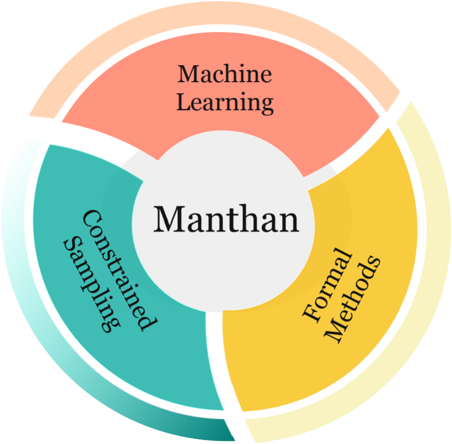
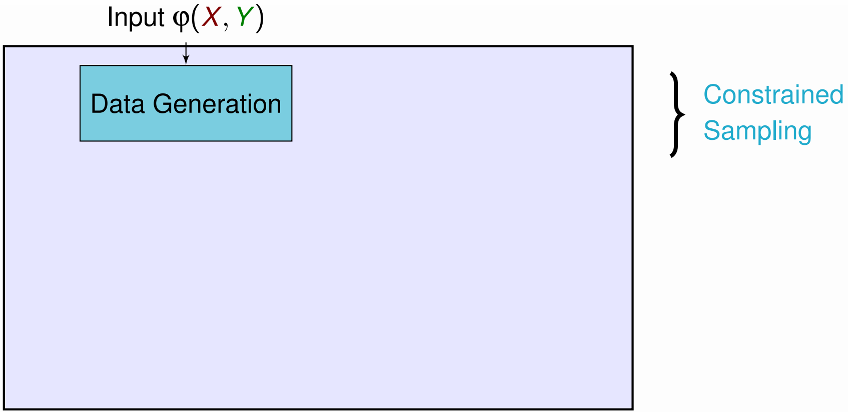
- Incremental determinization

  (Rabe et al., 2015, 2018, 2019)

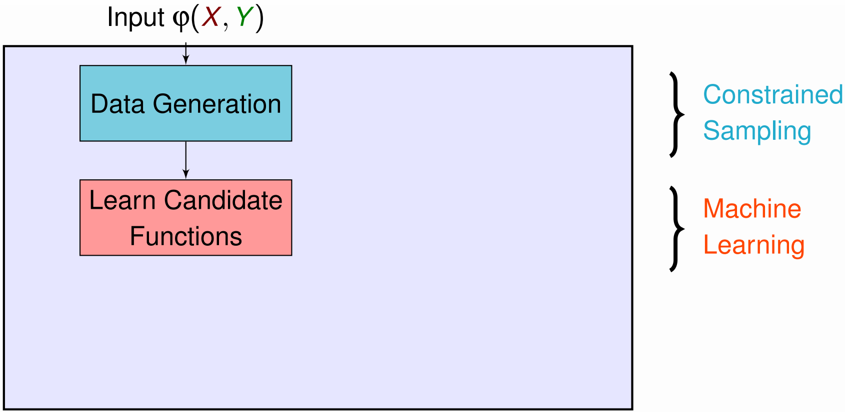
Scalability remains the holy grail

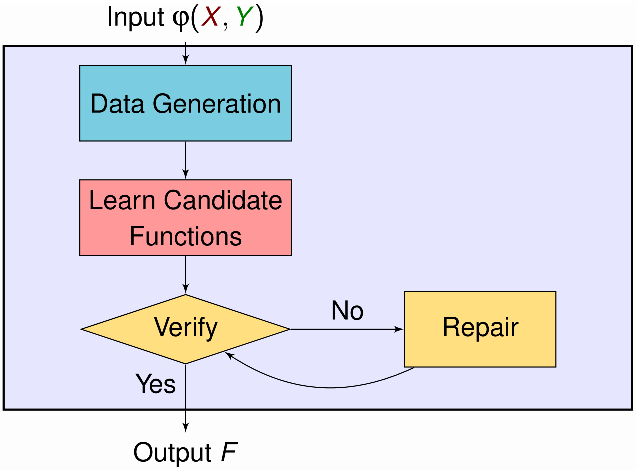
# A Data-Driven Approach for Boolean Functional Synthesis




Input φ(X, Y)
Data Generation
Constrained Sampling

# Manthan


Input $\varphi(X, Y)$
Data Generation
Learn Candidate Functions
Constrained Sampling
Machine Learning




Constrained Sampling

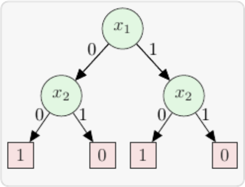
Machine Learning

Formal Methods

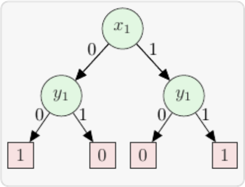
$$\varphi(x_1, x_2, y_1, y_2) \longrightarrow$$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |




$p_1 := (\neg x_1 \land \neg x_2)$,
$p_2 := (x_1 \land \neg x_2)$
$f_1$ = if $p_1$ then 1
elif $p_2$ then 1
else 0

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$p_2 := (x_1 \land y_1)$
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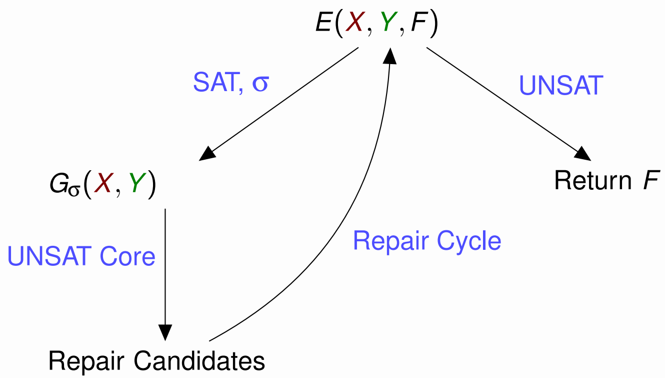
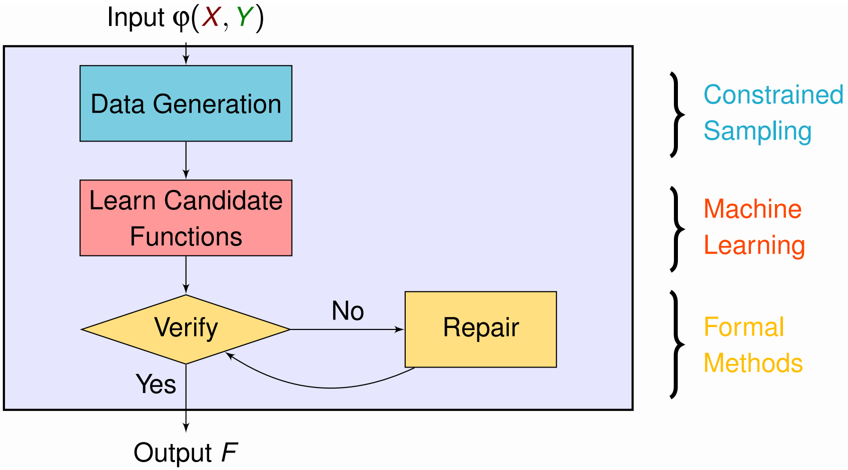
else 0

# Repair of Approximations

## Reaping the Fruits of Formal Methods Revolution




Input φ(X, Y)
Data Generation
Learn Candidate Functions
Verify
No
Repair
Yes
Output F
Constrained Sampling
Machine Learning
Formal Methods

# Data Generation

Potential Strategy: Randomly sample satisfying assignment of $\varphi(X, Y)$.

Challenge: Multiple valuations of $y_1, y_2$ for same valuation of $x_1, x_2$.

# Data Generation

Potential Strategy: Randomly sample satisfying assignment of $\varphi(X, Y)$.

Challenge: Multiple valuations of $y_1, y_2$ for same valuation of $x_1, x_2$.

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0/1 |
| 0 | 1 | 0/1 | 0/1 |
| 1 | 0 | 0/1 | 0/1 |
| 1 | 1 | 0/1 | 0 |

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0/1 |
| 0 | 1 | 0/1 | 0/1 |
| 1 | 0 | 0/1 | 0/1 |
| 1 | 1 | 0/1 | 0 |

$\xrightarrow{\text{Uniform Sampler}}$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0/1 |
| 0 | 1 | 0/1 | 0/1 |
| 1 | 0 | 0/1 | 0/1 |
| 1 | 1 | 0/1 | 0 |

$\xrightarrow{\text{Uniform Sampler}}$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

- Possible Skolem functions:
  - $f_1(x_1, x_2) = \neg(x_1 \vee x_2)$
  - $f_2(x_1, x_2) = \neg(x_1 \wedge x_2)$

# Data Generation

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0/1 |
| 0 | 1 | 0/1 | 0/1 |
| 1 | 0 | 0/1 | 0/1 |
| 1 | 1 | 0/1 | 0 |

$\xrightarrow{\text{Uniform Sampler}}$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 |

- Possible Skolem functions:
  - $f_1(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad f_1(x_1, x_2) = \neg x_1$ $\quad f_1(x_1, x_2) = \neg x_2$ $\quad f_1(x_1, x_2) = 1$
  - $f_2(x_1, x_2) = \neg(x_1 \wedge x_2)$ $\quad f_2(x_1, x_2) = \neg x_1$ $\quad f_2(x_1, x_2) = \neg x_2$ $\quad f_2(x_1, x_2) = 0$

# Data Generation

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \vee x_2 \vee y_1) \wedge (\neg x_1 \vee \neg x_2 \vee \neg y_2)$$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0/1 |
| 0 | 1 | 0/1 | 0/1 |
| 1 | 0 | 0/1 | 0/1 |
| 1 | 1 | 0/1 | 0 |

$\xrightarrow{\text{Magical Sampler}}$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

- Possible Skolem functions:
  - $f_1(x_1, x_2) = \neg(x_1 \vee x_2)$ $\quad f_1(x_1, x_2) = \neg x_1$ $\quad f_1(x_1, x_2) = \neg x_2$ $\quad f_1(x_1, x_2) = 1$
  - $f_2(x_1, x_2) = \neg(x_1 \wedge x_2)$ $\quad f_2(x_1, x_2) = \neg x_1$ $\quad f_2(x_1, x_2) = \neg x_2$ $\quad f_2(x_1, x_2) = 0$

# Weighted Sampling to Rescue

- $W : X \cup Y \mapsto [0, 1]$

- The probability of getting an assignment is proportional to its weight.

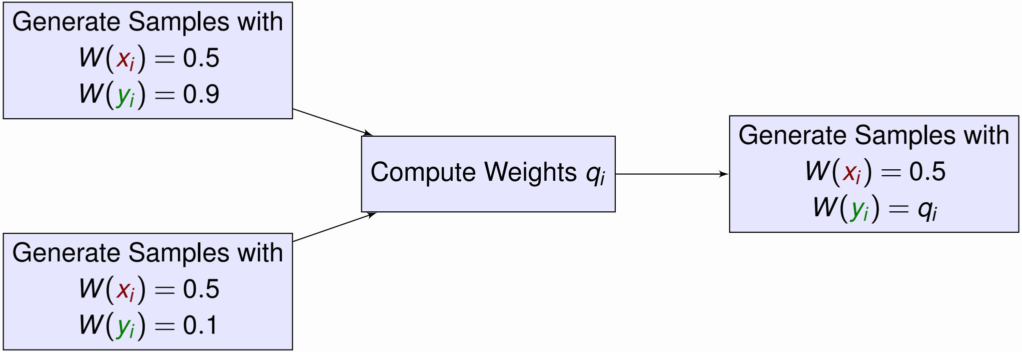
$$W(\sigma) = \prod_{\sigma(z_i)=1} W(z_i) \prod_{\sigma(z_i)=0} (1 - W(z_i))$$

- Example: $W(x_1) = 0.5 \quad W(x_2) = 0.5 \quad W(y_1) = 0.9 \quad W(y_2) = 0.1$
  $\sigma_1 = \{x_1 \mapsto 1, x_2 \mapsto 0, y_1 \mapsto 0, y_2 \mapsto 1\}$

$$W(\sigma_1) = 0.5 \times (1 - 0.5) \times (1 - 0.9) \times 0.1 = 0.0025$$

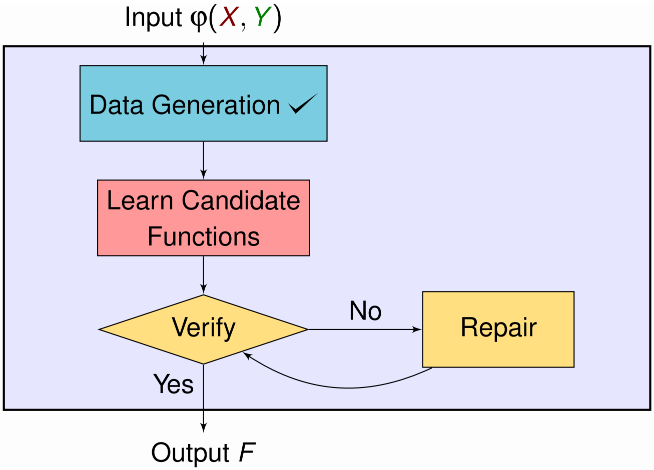
- Uniform sampling is a special case where all variables are assigned weight of 0.5.

# Data Generation



# Different Sampling Strategies

- Knowledge representation based techniques
  - (Yuan,Shultz, Pixley,Miller,Aziz 1999)
  - (Yuan,Aziz, Pixley,Albin, 2004)
  - (Kukula and Shiple, 2000)
  - (Sharma, Gupta, M., Roy, 2018)
  - (Gupta, Sharma, M., Roy, 2019)
- Hashing based techniques
  - (Chakraborty, M., and Vardi 2013, 2014,2015)
  - (Soos, M., and Gocht 2020)
- Mutation based techniques
  - (Dutra, Laeufer, Bachrach, Sen, 2018)
- Markov Chain Monte Carlo based techniques
  - (Wei and Selman,2005)
  - ( Kitchen,2010)
- Constraint solver based techniques
  - (Ermon, Gomes, Sabharwal, Selman,2012)
- Belief networks based techniques
  - (Dechter, Kask, Bin, Emek,2002)
  - ( Gogate and Dechter,2006)


Input φ(X, Y)
Data Generation ✓
Learn Candidate Functions
Verify
No
Repair
Yes
Output F

# Learn Candidate Function: Decision Tree Classifier

$$\varphi(x_1, x_2, y_1, y_2) : (x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$$

- To learn $y_2$
  - Feature set: valuation of $x_1, x_2, y_1$
  - Label: valuation of $y_2$
  - Learn decision tree to represent $y_2$ in terms of $x_1, x_2, y_1$
- To learn $y_1$
  - Feature set: valuation of $x_1, x_2$
  - Label: valuation of $y_1$
  - Learn decision tree to represent $y_1$ in terms of $x_1, x_2$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
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# Learning Candidate Functions

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
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| 0 | 1 | 0 | 1 |
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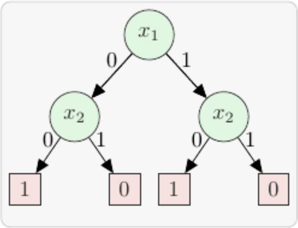


$p_1 := (\neg x_1 \wedge \neg x_2)$,
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$f_1$ = if $p_1$ then 1
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else 0

# Verification of Candidate Functions

$$E(X,Y,Y') := \varphi(X,Y) \land \neg\varphi(X,Y') \land (Y' \leftrightarrow F(X))$$

(JSCTA'15)

- If $E(X,Y,Y')$ is UNSAT: $\exists Y \varphi(X,Y) \equiv \varphi(X,F(X))$
  - Return $F$
- If $E(X,Y,Y')$ is SAT: $\exists Y \varphi(X,Y) \not\equiv \varphi(X,F(X))$
  - Let $\sigma \models E(X,Y,Y')$ be a counterexample to fix.

# Repair Candidate Identification

$$E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$$

$\sigma \models E(X, Y, Y')$ be a counterexample to fix.

- Let $\sigma := \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 1, y_2 \mapsto 1, y'_1 \mapsto 0, y'_2 \mapsto 0\}$.
- Potential repair candidates: All $y_i$ where $\sigma[y_i] \neq \sigma[y'_i]$.

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- Potential repair candidates: All $y_i$ where $\sigma[y_i] \neq \sigma[y'_i]$.
- $\varphi(X, Y)$ is Boolean Relation.
  - So it can be $\hat{\sigma} = \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 0, y_2 \mapsto 1, y'_1 \mapsto 0, y'_2 \mapsto 0\}$
  - We would not repair $f_1$.

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- Let $\sigma := \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 1, y_2 \mapsto 1, y_1' \mapsto 0, y_2' \mapsto 0\}$.
- Potential repair candidates: All $y_i$ where $\sigma[y_i] \neq \sigma[y_i']$.
- $\varphi(X, Y)$ is Boolean Relation.
  - So it can be $\hat{\sigma} = \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 0, y_2 \mapsto 1, y_1' \mapsto 0, y_2' \mapsto 0\}$
  - We would not repair $f_1$.
- MaxSAT-based Identification of *nice counterexamples*:
  - Hard Clauses $\varphi(X, Y) \land (X \leftrightarrow \sigma[X])$.
  - Soft Clauses $(Y \leftrightarrow \sigma[Y'])$.
- Candidates to repair: Y variables in the violated soft clauses

## Repairing Approximations

- $\sigma = \{x_1 \mapsto 1, x_2 \mapsto 1, y_1 \mapsto 0, y_2 \mapsto 1, y_1' \mapsto 0, y_2' \mapsto 0\}$, and we want to repair $f_2$.
- Potential Repair: If $\underbrace{x_1 \wedge x_2 \wedge \neg y_1}_{\beta = \{x_1, x_2, \neg y_1\}}$ then $y_2 = 1$
- Would be nice to have $\beta = \{x_1, x_2\}$ or even $\beta = \{x_1\}$
- Challenge: How do we find small $\beta$?
  - $G_\sigma(X, Y) := \varphi(X, Y) \wedge x_1 \wedge x_2 \wedge \neg y_1 \wedge (y_2 = 0)$
  - $\beta$:= Literals in UNSAT Core of $G_\sigma(X, Y)$

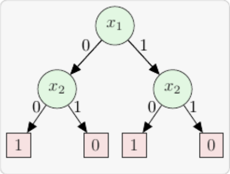
# Manthan

$\varphi(X, Y)$
$X = \{x_1, x_2\}$
$Y = \{y_1, y_2\}$

Data Generation

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

Learn Candidates




Verify Candidates

Check Satisfiability of $E(X, Y, Y')$

SAT, $\sigma$

$G_\sigma(X, Y)$

UNSAT Core-based Repair

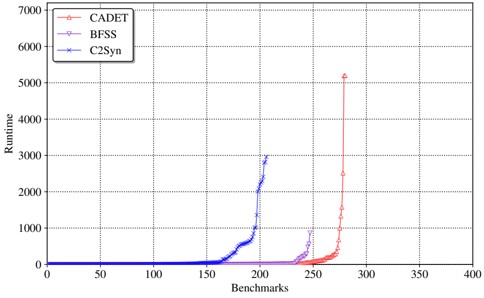
UNSAT

Return F

# Experimental Evaluations

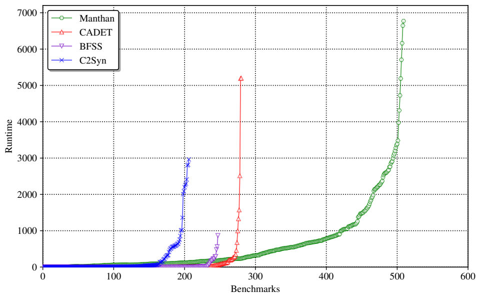
- 609 Benchmarks from:
  - QBFEval competition
  - Arithmetic
  - Disjunctive decomposition
  - Factorization
- Compared Manthan with State-of-the-art tools: CADET ( Rabe et al., 2019 ), BFSS (Akshay et al. ,2018), C2Syn (Chakraborty et al., 2019).
- Timeout: 7200 seconds.

# Experimental Evaluations




| C2Syn | BFSS | CADET |
|---|---|---|
| 206 | 247 | 280 |

# Experimental Evaluations




| C2Syn | BFSS | CADET | Manthan |
|---|---|---|---|
| 206 | 247 | 280 | 509 |

An increase of 223 benchmarks.

# Future work: Interesting Questions

- Learning Theoretic Foundations for Functional Synthesis
  - What is the ideal distribution to generate the data?
  - Mistake bounds/complexity of learning functions from relations?
- The Future of Formal Methods (FM) +Machine Learning (ML)
  - The proposed solutions by ML do not need to be fully correct.
  - Use FM for correctness and ML to quickly find the solution.

# Conclusion

Manthan: A Data-Driven Approach for Boolean Functional Synthesis.

- Constrained Sampling
- Decision List Classifier
- Formal Methods

Solves 509 benchmarks — state of the art could solve 280

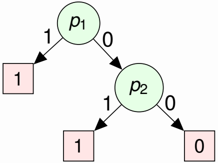
https://github.com/meelgroup/manthan

Thanks!

# Repair: Adding Level to Decision List

- Candidates are from one level decision list:
  - Say we have paths $p_1, p_2$ with the leaf node label as 1.
  - Learned decision tree: If $p_1$ then 1, elif $p_2$ then 1, else 0.
  - $p_1$, $p_2$ can be reordered.

Can reorder $p_1, p_2$



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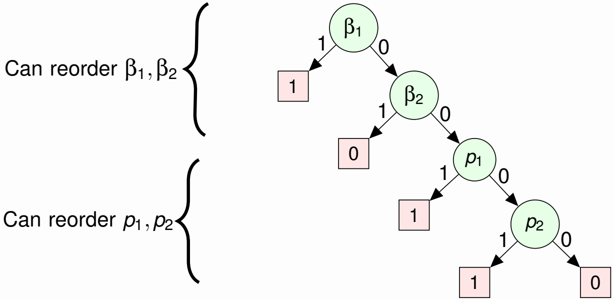



- Suppose in repair iterations, we have learned: If $\beta_1$ then 1, $\ldots \beta_2$ then 0

  ......

- $\beta_1$ and $\beta_2$ can be reordered.

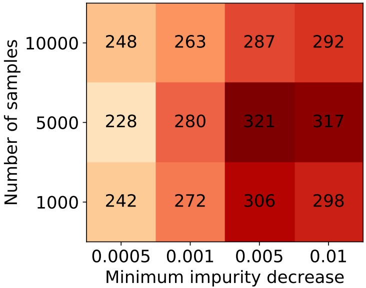
- From one level decision list to two decision list.

# Data Generation: Experimental Evaluations(II)

Impact of different sampling schemes and the quality of samplers.

| Sampler | Instances Solved with No Repair | Total Instances Solved |
|---|---|---|
| CryptoMiniSAT | 14 | 271 |
| QuickSampler | 28 | 275 |
| Uniform Sampler | 51 | 345 |
| Weighted Sampler | 66 | 356 |

# Learning Candidate Functions: Experimental Evaluations(I)




- Learning without any errors on sampled data: Manthan could only solves 162 instances.
- Manthan decides the number of sampler as per cardinality of $Y$ variables, and uses 0.005 as minimum impurity decrease parameter.

# Manthan: Example

- Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$
- $\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$
- Skolem Functions:
  - $f_1(x_1, x_2) := (x_1 \vee x_2)$
  - $f_2(x_1, x_2, y_1) := (x_1 \wedge (x_2 \vee y_1))$
    $f_2(x_1, x_2, y_1) := (x_1 \wedge (x_2 \vee (x_1 \vee x_2))$
    $f_2(x_1, x_2, y_1) := x_1$

$$\exists Y \varphi(X, Y) \equiv \varphi(X, F(X))$$

# Example: Data Generation

Let $X = \{x_1, x_2\}$, and $Y = \{y_1, y_2\}$

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

$$\Big\downarrow \text{Constrained Sampler}$$

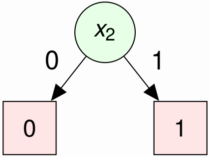
| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |

# Example: Learning Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- Learn candidate function $f_1$.
- Feature set for $y_1 := \{x_1, x_2\}$

| $x_1$ | $x_2$ | $y_1$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |



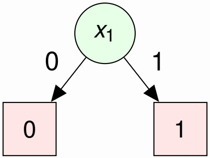

$$f_1(x_1, x_2) := x_2$$

# Example: Learning Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- Learn candidate function $f_2$.
- Feature set for $y_2$ := $\{x_1, x_2, y_1\}$

| $x_1$ | $x_2$ | $y_1$ | $y_2$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 |




$$f_2(x_1, x_2, y_1) := x_1$$

# Example: Verification of Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$

$$E(X, Y, Y') := \varphi(x_1, x_2, y_1, y_2) \wedge \neg\varphi(x_1, x_2, y_1', y_2') \wedge (y_1' \leftrightarrow x_2) \wedge (y_2' \leftrightarrow x_1)$$

$$\downarrow$$

SAT

$$\downarrow$$

$$\sigma \models E(X, Y, Y') \longrightarrow \sigma[x_1] = 1, \sigma[x_2] = 0$$

$$\swarrow \qquad \searrow$$

$$\sigma[y_1] = 1,\ \sigma[y_2] = 1 \qquad \sigma[y_1'] = 0,\ \sigma[y_2'] = 1$$

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$

$$E(X, Y, Y') := \varphi(x_1, x_2, y_1, y_2) \wedge \neg\varphi(x_1, x_2, y'_1, y'_2) \wedge (y'_1 \leftrightarrow x_2) \wedge (y'_2 \leftrightarrow x_1)$$

$$\downarrow$$

SAT

$$\downarrow$$

$$\sigma \models E(X, Y, Y') \longrightarrow \sigma[x_1] = 1, \sigma[x_2] = 0$$

$$\sigma[y_1 = 1], \sigma[y_2] = 1 \qquad \sigma[y'_1 = 0], \sigma[y'_2] = 1$$

$\sigma[y_1] \neq \sigma[y'_1]$

Candidate to repair $f_1$

## Example: Repairing candidate functions (I)

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $G_1(X, Y) = \varphi(X, Y) \wedge (X \leftrightarrow \sigma[X]) \wedge (y_1 \leftrightarrow \sigma[y_1']]$.
- $G_1(X, Y) = \varphi(X, Y) \wedge (x_1 \leftrightarrow 1) \wedge (x_2 \leftrightarrow 0) \wedge (y_1 \leftrightarrow 0)$.
- UNSAT core of $G_1(X, Y) = \varphi(X, Y) \wedge (x_1 \leftrightarrow 1) \wedge (y_1 \leftrightarrow 0)$
- Repair formula $\beta = x_1$.

# Example: Repairing candidate functions (II)

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

| Before repair | Repair | After repair |
|---|---|---|
| $f_1(\sigma[X]) \mapsto 0$ | $f_1(X) \leftarrow f_1(X) \vee \beta$<br>$f_1(X) \leftarrow x_2 \vee x_1$ | $f_1(X) \mapsto 1$ |

# Example: Verification of Candidate Functions

$$\varphi(X, Y) := (y_1 \leftrightarrow (x_1 \vee x_2)) \wedge (y_2 \leftrightarrow (x_1 \wedge (x_2 \vee y_1)))$$

- $E(X, Y, Y') := \varphi(X, Y) \wedge \neg\varphi(X, Y') \wedge (Y' \leftrightarrow F(X))$

$$E(X, Y, Y') := \varphi(x_1, x_2, y_1, y_2) \wedge \neg\varphi(x_1, x_2, y'_1, y'_2) \wedge (y'_1 \leftrightarrow x_2 \vee x_1) \wedge (y'_2 \leftrightarrow x_1)$$

$$\downarrow$$

UNSAT

$$\downarrow$$

Manthan returns $F$

# Data Generation

- $\Sigma_1 :=$ Sample 500 data point with $W(x_i) = 0.5$ and $W(y_i) = 0.9$.

$$w_1(i) = \frac{\text{Count}(\Sigma_1 \cap (y_i = 1))}{500}$$

- $\Sigma_2 :=$ Sample 500 data point with $W(x_i) = 0.5$ and $W(y_i) = 0.1$.

$$w_2(i) = \frac{\text{Count}(\Sigma_2 \cap (y_i = 0))}{500}$$

- If $0.35 < w_1(i) < 0.65$ and $0.35 < w_2(i) < 0.65$, then $q_i = w_1(i)$, else $q_i = 0.9$.