## Manthan: A Data-Driven Approach for Boolean **Functional Synthesis**

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## **Boolean Functional Synthesis**

- Given: A Boolean relation F(X, Y), with inputs  $X = \{x_1, ..., x_n\}$ , and outputs  $Y = \{y_1, ..., y_m\}$
- Problem Statement: Synthesise a function vector  $\Psi = \langle \psi_1 \dots \psi_m \rangle$ , where  $\psi_i$  is a function for variable  $y_i$ ,

such that

- $y_i = \psi_i(x_1, \ldots, x_n)$
- $\exists YF(X, Y) \equiv F(X, \Psi(X))$

• Each  $\psi_i$  is called Skolem function, and  $\Psi$  is called Skolem function vector.

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#### $F(X, Y) : x_1 \lor x_2 \lor y_1$

 $x_1 \lor x_2 \lor y_1 \longrightarrow \neg(x_1)$ 

$$\neg (x_1 \lor x_2) \rightarrow y_1$$

$$\downarrow$$

$$\psi_1(x_1, x_2) = \neg (x_1 \lor x_2)$$

# Skolem Function: Example Let $X = \{x_1, x_2\}$ and $Y = \{y_1\}$ $F(X, Y) : x_1 \lor x_2 \lor y_1$ Skolem functions: $x_1 \lor x_2 \lor y_1 \longrightarrow \neg(x_1)$ $\psi_1(z)$ $F(X, \Psi(X)) = x_1 \lor x_2 \lor \neg (x_1 \lor x_2)$

$$\begin{array}{c} (x_1, x_2) \rightarrow y_1 \\ \downarrow \\ x_1, x_2) = \neg (x_1 \lor x_2) \end{array}$$

# Skolem Function: Example Let $X = \{x_1, x_2\}$ and $Y = \{y_1\}$ $F(X, Y) : x_1 \lor x_2 \lor y_1$ Skolem functions: $x_1 \lor x_2 \lor y_1 \longrightarrow \neg(x_1)$ $\psi_1(y)$ $F(X, \Psi(X)) = x_1 \lor x_2 \lor \neg (x_1 \lor x_2)$

 $\exists YF(X, Y) \equiv F(X, \Psi(X))$ 

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$$, x_2) = \neg x_2$$

$$(x_2) = 1$$

## Applications

- Application in a wide variety of domains:
  - Certified QBF solving
  - Program synthesis
  - Cryptography

• Given a *n*-bit number *X*, find *m*-bit  $Y \notin \{1,X\}$ .

 $Y \notin \{1,X\}.$ 

• As a relation between input and output values

 $F(X, Y) : \frac{X}{Y} \in \mathbb{Z} \text{ and } Y \notin \{1, X\}$ 

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• As a relation between input and output values



Boolean relation F

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**Boolean relation F** 

#### • Given a *n*-bit number X, find *m*-bit number Y such that Y divides X and

Ψ

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#### George Boole

## State of the Art



#### **Thoralf Albert Skolem**



**George Boole** 

• Theoretically

must take super-polynomial time. (Akshay *et al.*,2018)

## State of the Art



**Thoralf Albert Skolem** 

# Unless P = NP, there exist problem instance where Boolean function synthesis

- From the proof of validity of  $\forall X \exists YF(X, Y)$ Bendetti et al, 2005 Jussilla et al, 2007 Heule et al, 2014
- Quantifier instantiation in SMT solvers Barrett et al, 2015 Bierre et al,2017
- Input-Output separation: Chakraborty et al., 2018

## State of the Art

- Knowledge representation: Kukula et al, 2000 Trivedi et al, 2003 Jiang, 2009 Kuncak et al., 2010 Balabanov and Jiang, 2011 John et al., 2015 Fried, Tabajara, Vardi, 2016,2017 Akshay et al., 2017,2018 Chakraborty et al., 2019
- Incremental determinization: Rabe et. al, 2015, 2018, 2019



#### A Data-Driven Approach for Boolean Functional Synthesis

#### Learn Candidate Functions

#### **Repair Candidate Functions**

## Manthan



## Data Generation
• Satisfying assignments of *F*(*X*, *Y*): the valuations of *X* and *Y* that satisfy the *F*(*X*, *Y*).

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But, the possible solution space is  $2^{|X|+|Y|}$ 

• Satisfying assignments of F(X, Y): the valuations of X and Y that satisfy the F(X, Y).

> But, the possible solution space is  $2^{|X|+|Y|}$

• Sample uniformly at random from the solution space of satisfying assignments.

#### • We want to capture the relation between *X* and *Y*.

 $F(x_1, x_2, y_1, y_2)$ :  $(x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$ 

<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
Ο	0	1	0/1
Ο	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	Ο

#### • We want to capture the relation between X and Y.

 $F(x_1, x_2, y_1, y_2)$ :  $(x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$ 

$x_1$	<i>x</i> <sub>2</sub>	<i>Y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
Ο	Ο	1	0/1
Ο	1	0/1	0/1
1	Ο	0/1	0/1
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Unlike classical machine leaning for the same valuation of  $x_1, x_2$ : different  $y_1$ 

•  $F(x_1, x_2, y_1, y_2)$ :  $(x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$ 

<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
0	0	1	0/1
0	1	0/1	0/1
1	0	0/1	0/1
1	1	0/1	0

Sample 4 data points uniformly at random

<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	0

•  $F(x_1, x_2, y_1, y_2)$ :  $(x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$ 

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Sample 4 data points uniformly at random

• Possible Skolem function

$$\psi_1(x_1, x_2) = \neg (x_1 \lor x_2) \qquad \psi_1(x_1, x_2) = \neg x_1 \qquad \psi_1(x_1, x_2) = \neg x_2 \qquad \psi_1(x_1, x_2) = 1$$
  
$$\psi_2(x_1, x_2) = \neg (x_1 \lor x_2) \qquad \psi_2(x_1, x_2) = \neg x_1 \qquad \psi_2(x_1, x_2) = \neg x_2 \qquad \psi_2(x_1, x_2) = 0$$

<i>x</i> <sub>1</sub>	<i>x</i> <sub>2</sub>	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
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•  $F(x_1, x_2, y_1, y_2)$ :  $(x_1 \lor x_2 \lor y_1) \land (\neg x_1 \lor \neg x_2 \lor \neg y_2)$ 

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• Possible Skolem function

$$\begin{aligned} \psi_1(x_1, x_2) &= \neg (x_1 \lor x_2) & \psi_1(x_1, x_2) = \neg x_1 & \psi_1(x_1, x_2) = \neg x_2 & \psi_1(x_1, x_2) = 1 \\ \psi_2(x_1, x_2) &= \neg (x_1 \lor x_2) & \psi_2(x_1, x_2) = \neg x_1 & \psi_2(x_1, x_2) = \neg x_2 & \psi_2(x_1, x_2) = 0 \end{aligned}$$

Sample 4 data points

$x_1$	$x_2$	<i>y</i> <sub>1</sub>	<i>y</i> <sub>2</sub>
0	0	1	0
0	1	1	0
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1	1	1	0

while biasing the valuation of *Y*.



#### • We design a weighted sampling strategy that seeks to uniformly sample X,

Find Bias q for Y

X: Uniform sampling Y: Sampling with bias q



•  $F(x_1, x_2, y_1)$  :  $x_1 \lor x_2 \lor y_1$ 

•  $F(x_1, x_2, y_1)$  :  $x_1 \lor x_2 \lor y_1$ 

<i>X</i> <sub>1</sub>	$X_2$	У <sub>1</sub>
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- $F(x_1, x_2, y_1)$ :  $x_1 \lor x_2 \lor y_1$ 
  - Feature set: valuation of  $x_1, x_2$
  - Label: valuation of *y*<sub>1</sub>
  - Learn decision tree to represent  $y_1$  in terms of  $x_1, x_2$ .

$X_1$	$X_2$	У <sub>1</sub>	
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  - Feature set: valuation of  $x_1, x_2$
  - Label: valuation of y<sub>1</sub>
  - Learn decision tree to represent  $y_1$  in terms of  $x_1, x_2$ .

- To learn  $\psi_i$ :
  - Feature set: valuation of X in data
  - Label: valuation of y<sub>i</sub> in data
  - Learn decision tree classifier



#### **Binary classification** problem.



• When Candidate functions are not Skolem functions:

#### $\exists YF(X, Y) \not\equiv F(X, \Psi(X))$

There exists at least one valuation of X where  $\exists YF(X, Y)$  evaluates to True, and  $F(X, \Psi(X))$  also evaluates to False.

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• When Candidate functions are Skolem functions:

also evaluates to True.

- $\exists YF(X, Y) \not\equiv F(X, \Psi(X))$
- There exists at least one valuation of X where  $\exists YF(X, Y)$  evaluates to True,

- $\exists YF(X, Y) \equiv F(X, \Psi(X))$
- For all the valuation of X where  $\exists YF(X, Y)$  evaluates to True,  $F(X, \Psi(X))$

#### $E(X, Y, Y') = F(X, Y) \land \neg F(X, Y') \land (Y' \leftrightarrow \Psi)$

Y and *Y*' are different, but same X

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If there exists a valuation of X, s.t. F(X, Y) evaluates to True, and F(X, Y')evaluates to False



• Check satisfiability of E(X, Y, Y').

• If E(X, Y, Y') is UNSAT: return the Skolem function  $\Psi$ .

• If E(X, Y, Y') is SAT: let  $\sigma \models E(X, Y, Y')$  be a counter example to fix.

 $E(X, Y, Y') = F(X, Y) \land \neg F(X, Y') \land (Y' \leftrightarrow \Psi)$ 

# **Repairing Candidate Functions**

• E(X, Y, Y') is SAT:  $\sigma \models E(X, Y, Y')$ 



 $E(X, Y, Y') = F(X, Y) \land \neg F(X, Y') \land (Y' \leftrightarrow \Psi)$ 

• The potential candidates to repair : functions corresponding to  $y_i$ , if  $\sigma[y_i] \neq \sigma[y'_i]$ 

# **Repairing Candidate Functions**

- $(X \leftrightarrow \sigma[X]).$
- Let candidate function  $\psi_i(X)$  corresponding to  $y_i$  needs to repair. With  $X \leftrightarrow \sigma[X]$ Before repair  $\psi_i(X) \mapsto 0$  $\psi_i(X) \mapsto 1$

• The aim of a repair iteration is to make  $F(X, Y') \land (Y' \leftrightarrow \Psi)$  evaluates to True with

After repair

 $\psi_i(X) \mapsto 1$ 

 $\psi_i(X) \mapsto 0$ 

# **Repairing Candidate Functions**

- Find UNSAT core of  $G_i(X, Y) = F(X, Y) \land (X \leftrightarrow \sigma[X]) \land (y_i \leftrightarrow \sigma[y_i'])$
- Use the UNSAT core to construct  $\beta$  repair formula With  $X \leftrightarrow \sigma[X]$

Before repair	Repair	After repair
$\psi_i(X) \mapsto 0$	$\psi_i(X) \leftarrow \psi_i(X) \lor \beta$	$\psi_i(X) \mapsto 1$
$\psi_i(X) \mapsto 1$	$\psi_i(X) \leftarrow \psi_i(X) \land \neg \beta$	$\psi_i(X) \mapsto 0$



#### Manthan



# **Experimental Evaluations**

- 609 Benchmarks from:
  - QBFEval competition 2-QBF track
  - Arithmetic set (Fried, Tabajara, Vardi, 2016)
  - Disjunctive decomposition set (Akshay et al., 2017)
  - Factorization set (Akshay et al., 2017)
- Compared Manthan with State-of-the-art tools: CADET (Rabe et. al, 2019), BFSS (Akshay et al., 2018), C2Syn (Chakraborty et al., 2019).
- Timeout: 7200s

#### **Experimental Evaluations**


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DET	Manthan
80	356
26	

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DET	Manthan
80	356
26	1

#### An increase of 76 benchmarks

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DET	Manthan
80	356
26	

#### An increase of 76 benchmarks

Manthan  $\setminus$  All tools: 60

# Future work: interesting questions

- What is the ideal distribution to generate the data?
- How good are the candidate functions generated by data?

• Can we use similar approach for program synthesis, program repair ?

### Conclusion

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Constrained Sampling



**Constrained Sampling** 





Constrained Sampling





Constrained Sampling







Constrained Sampling







**Constrained Sampling** 









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### Conclusion



Solves 356 benchmarks — state of the art could solve 280.



**Constrained Sampling** 





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Opens up several interesting directions



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https://github.com/meelgroup/manthan





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Thanks !



